Portfolio Risk Management via a CVaR and Stochastic Dominance Hybrid Approach

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***Abstract*—Risk management is a crucial aspect in decision making under uncertainty. This paper proposes a hybrid risk management approach for stochastic programming-based portfolio optimization problems. The proposed approach consolidates the classical Markowitz mean-conditional value at risk (CVaR) model, which is the most commonly used approach in power and energy applications, and the stochastic dominance (SD) concept. A case study on the risk management of a wind power producer's stochastic bidding strategy in the electricity market is performed to demonstrate the superiority of the proposed approach over the commonly used mean-CVaR and the SD-based risk management approaches.**

***Index Terms*--Conditional value at risk (CVaR), portfolio optimization, risk management, stochastic dominance (SD), stochastic programming.**

I. Introduction

Problems of decision-making with uncertainties are common in different areas, such as economics, finance, and engineering, e.g., decision making of electricity market participants with uncertain information about market clearing price, demand, etc. A rational decision-making model should account for those uncertainties when making/recommending decisions. Such problems can be modelled using the stochastic programming approach [1], with the uncertain input parameters represented as scenarios, which are the plausible sets of input parameter values with associated probabilities of occurrence. A common approach to formulating a stochastic programming problem was to maximize the expected value of the objective's distribution (i.e., portfolio). However, this approach does not account for the objective's risk, which is defined to be the possibility that the value of the realized objective deviates adversely from what is expected.

Risk management was first proposed by Markowitz in his mean-variance stochastic programming model [2], [3], which optimizes the expected value of the objective's distribution while minimizing the risk associated with the objective's distribution that is represented by its variance. Since then, it has been agreed that the performance of the objective's distribution should be measured in two dimensions: expected value and risk; and many risk measures, in addition to variance, have been proposed and used in the literature. The most commonly used risk measures are value at risk (VaR) and conditional value at risk (CVaR), which is recognized to be superior to VaR in stochastic optimization applications [4].

An alternative approach based on the stochastic dominance (SD) concept for risk management in stochastic programming was proposed in [5]. In that approach, SD constraints (SDCs) were added to the problem's set of constraints to impose a minimum tolerable "reference" distribution, which is called "benchmark distribution" or simply "benchmark". Those SDCs modify the problem's feasible region such that the problem's optimal distribution outperforms or dominates the benchmark imposed by the user. The SDCs can have different orders from the most restrict first order to infinite order. However, the second order is most applicable to describe the preferences of rational and risk-averse decision-makers [6]. The benchmark should be selected properly to avoid any problem infeasibility that may occur. The benchmark selection problem, which has been the major obstacle to using this approach in risk management, was addressed in [7].

The mean-CVaR model only optimizes the expected value of the objective's distribution and the defined CVaR tail. In contrast, the SD approach can manage different parameters of the objective's distribution, but cannot guarantee the optimality of the CVaR tail. To leverage the performance of these two approaches, this paper proposes a hybrid approach that consolidates CVaR and SD for the risk management in stochastic programming problems. By using the proposed approach, the trade-off between the objective's expected value and the $\text{CVaR}-\alpha$ tail is optimized subject to the added second-order SDCs that can manage other parameters of the objective's distribution directly, e.g., the worst scenario value. A case study on the risk management of a wind power producer's optimal bidding strategy in the electricity market is carried out to demonstrate the superior performance of the proposed hybrid risk management approach over the commonly used mean-CVaR approach and the SD approach.

The remainder of the paper is organized as follows. Section II presents the mathematical formulation of the proposed hybrid risk management approach. Section III presents a case study of the proposed approach for generating the optimal bidding strategy of a wind power producer in the electricity market in comparison with the mean-CVaR and the SD-based risk management approaches. Section IV concludes the paper.

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II. Proposed Hybrid Risk Management Approach

The proposed hybrid risk management approach, as illustrated in Fig. 1, can be applied for multistage linear stochastic programming problems. For the sake of simplicity, a general two-stage stochastic program is considered.

A. Two-Stage Risk-Neutral Problem

The two-stage risk-neutral stochastic programming problem can be expressed in the following general form [1]:

$$\underset{x,y(\omega)}{\text{Maximize}} \quad c^\top x + \sum_{\omega \in \Omega} \pi(\omega) q(\omega)^\top y(\omega) \tag{1}$$

Subject to:

$$Ax = b \tag{2}$$

$$T(\omega)x + W(\omega)y(\omega) = h(\omega) \qquad , \forall \omega \in \Omega \tag{3}$$

$$x \in X, y(\omega) \in Y \qquad , \forall \omega \in \Omega \tag{4}$$

where x and $y(\omega)$ are the vectors of the first- and second-stage decision variables, respectively; c, $q(\omega)$, b, $h(\omega)$, A, $T(\omega)$, and $W(\omega)$ are the known vectors and matrices; ω is the index of the problem's scenarios and belongs to the scenario set Ω; and $\pi(\omega)$ is the probability of Scenario ω.

B. Two-Stage Problem with Risk Management

By using the proposed hybrid approach to manage the risk of the problem (1)-(4), the problem is reformulated as:

$$\underset{x,y(\omega),\eta,s(\omega),n(\omega,v)}{\text{Maximize}} \quad (1-\beta)\left[c^\top x + \sum_{\omega \in \Omega} \pi(\omega) q(\omega)^\top y(\omega)\right] + \beta\left[\eta - \frac{1}{1-\alpha}\sum_{\omega \in \Omega} \pi(\omega) s(\omega)\right] \tag{5}$$

Subject to:

$$Ax = b \tag{6}$$

$$T(\omega)x + W(\omega)y(\omega) = h(\omega) \qquad , \forall \omega \in \Omega \tag{7}$$

$$x \in X, y(\omega) \in Y \qquad , \forall \omega \in \Omega \tag{8}$$

$$\eta - \left(c^\top x + q(\omega)^\top y(\omega)\right) \le s(\omega) \qquad , \forall \omega \in \Omega \tag{9}$$

$$s(\omega) \ge 0 \qquad , \forall \omega \in \Omega \tag{10}$$

$$k(v) - (c^\top x + q(\omega)^\top y(\omega)) \le n(\omega, v) \qquad , \forall \omega \in \Omega, \forall v \in \Upsilon \tag{11}$$

$$\sum_{\omega \in \Omega} \pi(\omega) n(\omega, v) \le \sum_{v' \in \Upsilon} \tau(v') \max(k(v) - k(v'), 0) \qquad , \forall v \in \Upsilon \tag{12}$$

$$n(\omega, v) \ge 0, \qquad \forall \omega \in \Omega, \forall v \in \Upsilon \tag{13}$$

where η and $s(\omega)$ are auxiliary decision variables related to the CVaR term; α is the per-unit confidence level in the range [0, 1] that defines the CVaR$-\alpha$ tail size to be managed; β is the risk-aversion parameter in the range [0, 1] that manages the trade-off between the expected value and the CVaR$-\alpha$ tail of the objective's distribution; $k(v)$ and $\tau(v)$ are the prefixed value and probability, respectively, of the benchmark's scenario v; Υ is the set of the benchmark distribution's scenarios; $n(\omega, v)$ is an auxiliary decision variable related to the second-order SDCs. Constraints (6)-(8) are equivalent to the risk-neutral problem's constraints (2)-(4). Constraints (9) and (10) linearize the CVaR term in the objective function (5) [8]. Constraints (11)-(13) are the second-order SDCs, which impose a predefined benchmark to be outperformed by the optimal objective's distribution. In the problem (5)-(13), the decision maker's risk preference is defined by α, β, and the benchmark distribution $\{(k(v), \tau(v)),\ \forall v \in \Upsilon\}$ whose values are determined subjectively, based on the risk preference of the decision maker, within their predetermined ranges.

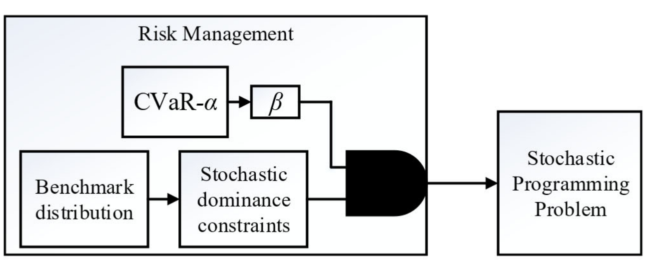

Fig. 1. Schematic diagram of the proposed CVaR and SD hybrid approach for the risk management in stochastic programming.

C. Proposed Hybrid Approach vs. Mean-CVaR Approach

The most common approach for the risk management in the stochastic programming problems in power system applications is the mean-CVaR approach, which is represented by the problem (5)-(10). Basically, the mean-CVaR approach has two objectives: maximizing the expected value of the objective's distribution and minimizing the CVaR$-\alpha$ tail of the objective's optimal distribution. However, the mean-CVaR approach does not manage directly other parameters of the optimal objective's distribution, such as the worst scenario. On the other hand, the second-order SDCs only guarantee that the optimal objective's distribution outperforms the imposed benchmark, but does not guarantee the minimization of the CVaR$-\alpha$ tail. The proposed hybrid approach combines the CVaR and SD approaches to facilitate the optimization of the expected value of the objective and the CVaR$-\alpha$ tail, while managing directly other parameters of the objective's distribution by satisfying a set of second-order SDCs. For instance, the worst scenario of the benchmark, imposed by the second-order SDCs, is a minimum limit that cannot be exceeded by the worst scenario of the optimal objective's distribution. Hence, the proposed approach has extra features over the mean-CVaR approach which could be crucial for the decision-makers in some applications.

III. A Case Study of Wind Power Bidding Problem

The optimal bidding problem of a wind power producer in a pool-based electricity market is used as an example to demonstrate the superior performance of the proposed hybrid risk management approach for stochastic programming problems. The wind power producer participates in the hourly day-ahead and balancing (or real-time) markets that are cleared 24 hours and one hour before the energy delivery, respectively. Any positive or negative generation deviation from the day-ahead market commitment should be covered through the participation in the balancing market.

The objective of the problem is to maximize the expected profit from trading in the day-ahead and balancing markets, while managing the risk caused by the three uncertainties considered: wind power generation, day-ahead market clearing price, and balancing market clearing price.

A. Bidding Model for the Wind Power Producer with the Proposed Hybrid Risk Management Approach

The bidding problem of the wind power producer that adopts the proposed hybrid risk management approach is formulated as follows:

$$\underset{W_{\omega}^{D},\eta,s(\omega),n(\omega,\nu)}{\text{Maximize}} (1-\beta)\{\sum_{\omega=1}^{N_{\Omega}} \pi(\omega)[\lambda_{\omega}^{D} W_{\omega}^{D} + \lambda_{\omega}^{r}(W_{\omega}^{ac} - W_{\omega}^{D})]\}$$
$$+\beta\,[\eta - \frac{1}{1-\alpha}\sum_{\omega=1}^{N_{\Omega}} \pi(\omega)\,.\,s(\omega)] \tag{14}$$

Subject to:

$$0 \le W_{\omega}^{D} \le W^{max} \quad , \forall\, \omega \tag{15}$$
$$(\lambda_{\omega}^{D} - \lambda_{\omega'}^{D})(W_{\omega}^{D} - W_{\omega'}^{D}) \ge 0 \quad , \forall \omega, \omega' \tag{16}$$
$$W_{\omega}^{D} = W_{\omega'}^{D}\,, \forall \omega, \omega' : \lambda_{\omega}^{D} = \lambda_{\omega'}^{D} \tag{17}$$
$$\eta - [\lambda_{\omega}^{D} W_{\omega}^{D} + \lambda_{\omega}^{r}(W_{\omega}^{ac} - W_{\omega}^{D})] \le s(\omega) \quad , \forall\, \omega \tag{18}$$
$$s(\omega) \ge 0 \quad , \forall\, \omega \tag{19}$$
$$k(\nu) - \pi(\omega)[\lambda_{\omega}^{D} W_{\omega}^{D} + \lambda_{\omega}^{r}(W_{\omega}^{ac} - W_{\omega}^{D})] \le n(\omega,\nu) \quad , \forall\, \omega, \forall\, \nu \tag{20}$$
$$\sum_{\omega=1}^{N_{\Omega}} \pi(\omega) n(\omega,\nu) \le \sum_{\nu'=1}^{N_{\nu'}} \tau(\nu').\max(k(\nu) - k(\nu'), 0) \quad , \forall\, \nu \tag{21}$$
$$n(\omega,\nu) \ge 0 \quad , \forall\, \omega, \forall\, \nu \tag{22}$$

where N_{Ω} is the number of problem's scenarios; λ_{ω}^{D} and λ_{ω}^{r} are the day-ahead and real-time clearing prices in Scenario ω, respectively; W_{ω}^{D} is the optimal power offer in the day-ahead market in Scenario ω; W_{ω}^{ac} is the actual wind power production in Scenario ω; and W^{max} is the maximum wind power generation capacity. The objective function (14) maximizes the expected revenue from trading in the day-ahead and balancing markets, while minimizing the CVaR$-\,\alpha$ tail of the objective's distribution. Constraint (15) limits the bidding capacity to the maximum wind power generation capacity. Constraint (16) forces a non-decreasing bidding curve. Constraint (17) represents the non-anticipativity conditions of the first-stage decisions. Constraints (18)-(19) are related to the CVaR term in the objective function (14). Constraints (20)-(22) are the second-order SDCs.

B. Data

A wind farm with the total installed capacity of 80 MW is considered. The historical data of wind power generation and market prices is obtained from the Southwest Power Pool (SPP) market. The autoregressive integrated moving average (ARIMA) model and the scenario generation and reduction methods [9] are applied to represent each random variable, including wind power generation, day-ahead price, and real-time price by five scenarios. MATLAB and Gurobi are used to code and solve the problem, respectively. All computations are carried out on a Windows-PC with a 3.4 GHz Core i7 CPU and 16 GB RAM, and the execution time was less than 10 seconds for each of the tried cases.

C. Results

The optimal bidding strategies of the wind power producer are obtained by using the risk-neutral model and the models with three different risk management approaches for six cases. Table I lists the model parameters of the six cases. Table II compares the parameters of interest for the optimal objectives' distributions' obtained from the different models listed in Table I for a typical hour. As the risk-neutral model does not

Table I: Values of risk management parameters for different models.

Cases			α	β	Benchmark				
						ν	1	2	3
1	Risk-neutral	Problem (14)-(19)	/	0	/				
2	Mean-CVaR		0.3	0.3					
3	SDCs	Problem (14)-(22)	/	/	B1	$\tau(\nu)$	1	/	/
4	CVaR+SD		0.3	0.3		$k(\nu)$	-1000	/	/
5	SDCs		/	/	B2	$\tau(\nu)$	0.05	0.6	0.35
6	CVaR+SD		0.3	0.3		$k(\nu)$	-800	0	200

Table II: Parameters of the optimal objectives' distributions obtained from different models in Table I.

	All in $	Expected value	Worst scenario	Expected value of negative tail	$CVaR-0.3$
1	Risk-neutral	854.17	-2784.6	-283.27	-269.43
2	Mean-CVaR	813	-2251	-83.2	0.39
3	SDCs, B1	674.6	-1000	-51.3	57.47
4	CVaR+SD, B1	663.2	-1000	-22.96	63.4
5	SDCs, B2	643.9	-800	-26.8	69.3
6	CVaR+SD, B2	635.8	-800	-14.64	74.84

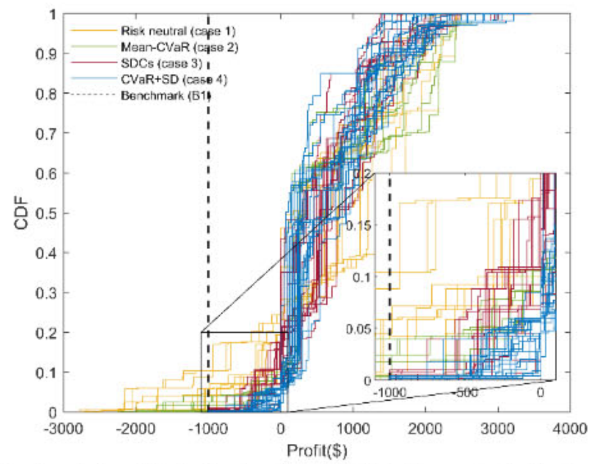

Fig. 2. Cumulative distribution functions (CDFs) of the optimal objectives' distributions for 24 hours of the cases 1, 2, 3, and 4, along with the CDF of the imposed benchmark B1.

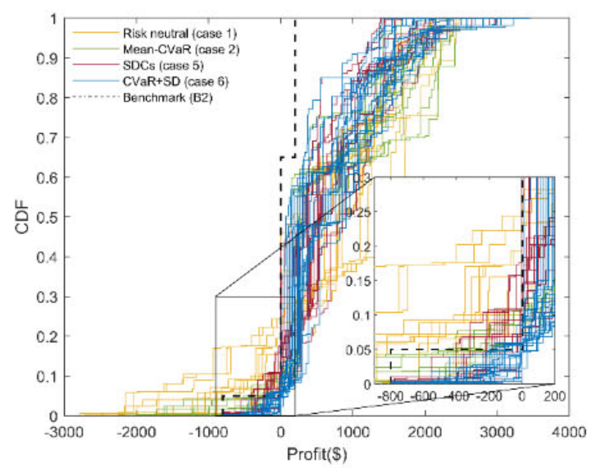

Fig. 3. CDFs of the optimal objectives' distributions for 24 hours of the cases 1, 2, 5, and 6, along with the CDF of the imposed benchmark B2.

consider risk management, it has the highest risk and the highest expected value as well. The worst scenarios of the objectives' distributions in the cases 3 to 6 that use SDCs are directly managed by the worst scenario of the corresponding benchmark imposed. Meanwhile, the expected value of the negative tail of the optimal objective's distribution is effectively managed in the cases 3 to 6 when using the SDCs. Moreover, compared to the cases 1, 3, 5, better CVaR–0.3 values are obtained in the cases 2, 4, and 6, respectively, when the CVaR is incorporated in the risk management.

To further demonstrate the risk management performance, the optimal objective functions' distributions of 24 hours of the six cases in Table I are shown in Figs. 2 and 3. As expected, for the cases 3, 4, 5 and 6, the worst scenarios of the optimal distributions do not exceed the worst scenario of the corresponding benchmark imposed. Moreover, the proposed approach effectively manages the negative and CVaR–0.3 tails simultaneously in the cases 4 and 6.

To further compare the three different risk management approaches, the effect of changing the values of the risk management parameters α and β and the benchmark on different parameters of the obtained optimal objective's distributions is studied. Different combinations of α = [0.05, 0.15, 0.25], β = [0:0.01:1], and a one-scenario benchmark with $k(1)$ = $[-2686{:}50{:}0]$ are applied to the corresponding risk management models. For each distribution of the obtained optimal objective, three parameters of interest are calculated: the expected value, the CVaR $-\alpha$ value, and the worst scenario's value. Fig. 4 depicts the values of these parameters of the obtained optimal objective's distributions versus the corresponding risk management parameters.

For the mean-CVaR model (the first row in Fig. 4), a small change in the value of β may lead to a significant change in the expected value and/or the worst scenario's value. For example, a 0.01 change in β's value leads to more than \$1000 change in the value of the worst scenario. Unlike the expected value and worst scenario's value that have abrupt changes, the CVaR$-\alpha$ value improves smoothly with the increase of the β value, except for the occurrence of some noticeable ripples due to the linearization of the CVaR term in (14). In contrast, in the SD-based model (the second row in Fig. 4), the smooth change of the one-scenario benchmark's value $k(1)$ leads to smooth changes in all of the three depicted parameters of interest. The $k(1)$ value and the worst scenario's value are always equal. However, the CVaR$-\alpha$ value does not change proportionally with the $k(1)$ value, which means that $k(1)$ value cannot be used directly to manage the CVaR$-\alpha$ value.

The proposed hybrid approach with a one-scenario benchmark optimizes the expected and CVaR$-\alpha$ values while managing directly the worst scenario's value. Hence, it can produce unique results of the optimal objective's distributions or combining the parameters of the optimal objective's distributions compared to those of the mean-CVaR and SD approaches. This is visually derivable from the 3D subfigures in Fig. 4 in which each colored surface corresponds to the output of the proposed hybrid approach. Meanwhile, the results of the mean-CVaR model are those in the vertical cross-section at the lowest value of $k(1)$ (i.e., -2686); and the results of the SD-based model are those in the vertical cross-section at $\beta = 0$. In the subfigures of expected value and CVaR$-\alpha$, the surfaces cover the values that are not covered in any of the corresponding vertical cross-sections. Again, the ripples/spikes in the CVaR $-\alpha$ subfigures are due to the linearization of its term in (14). For the worst scenario subfigures, the covered set of values by each surface is equal to the combination of the values' sets in the corresponding vertical cross-sections. Hence, the unique combinations of the expected, CVaR$-\alpha$, and worst scenario values are obtainable by the proposed approach but cannot be obtained by the mean-CVaR or the SD-based model. Such unique combinations cause disruptions/perturbations to the problem's efficient frontier, defined based on the mean-CVaR model or the SD-based model, by introducing new points or outperforming existing points.

Thus, by flexibly shaping the objective's distribution and managing different parameters of the objective's distribution, the proposed approach is superior to the mean-CVaR and the SD approaches for portfolio risk management.

IV. Conclusion

This paper proposed a hybrid risk management approach for portfolio optimization with uncertainty that was modeled as a stochastic programming problem. The proposed approach consolidates the commonly used mean-CVaR model and the SD-based model to leverage their performance simultaneously and achieve a superior risk management performance. The proposed approach enabled the optimization of the expected value of the objective and the CVaR- α tail, while managing directly other parameters of the objective's distribution (e.g., worst scenario value) specified through the benchmark imposed by a set of second-order SDCs. The optimal bidding problem of a wind power producer in the electricity market was used as an example to demonstrate the performance of the proposed approach in comparison with the mean-CVaR and SD-based models. Results showed the superior performance of the proposed hybrid approach in risk management.

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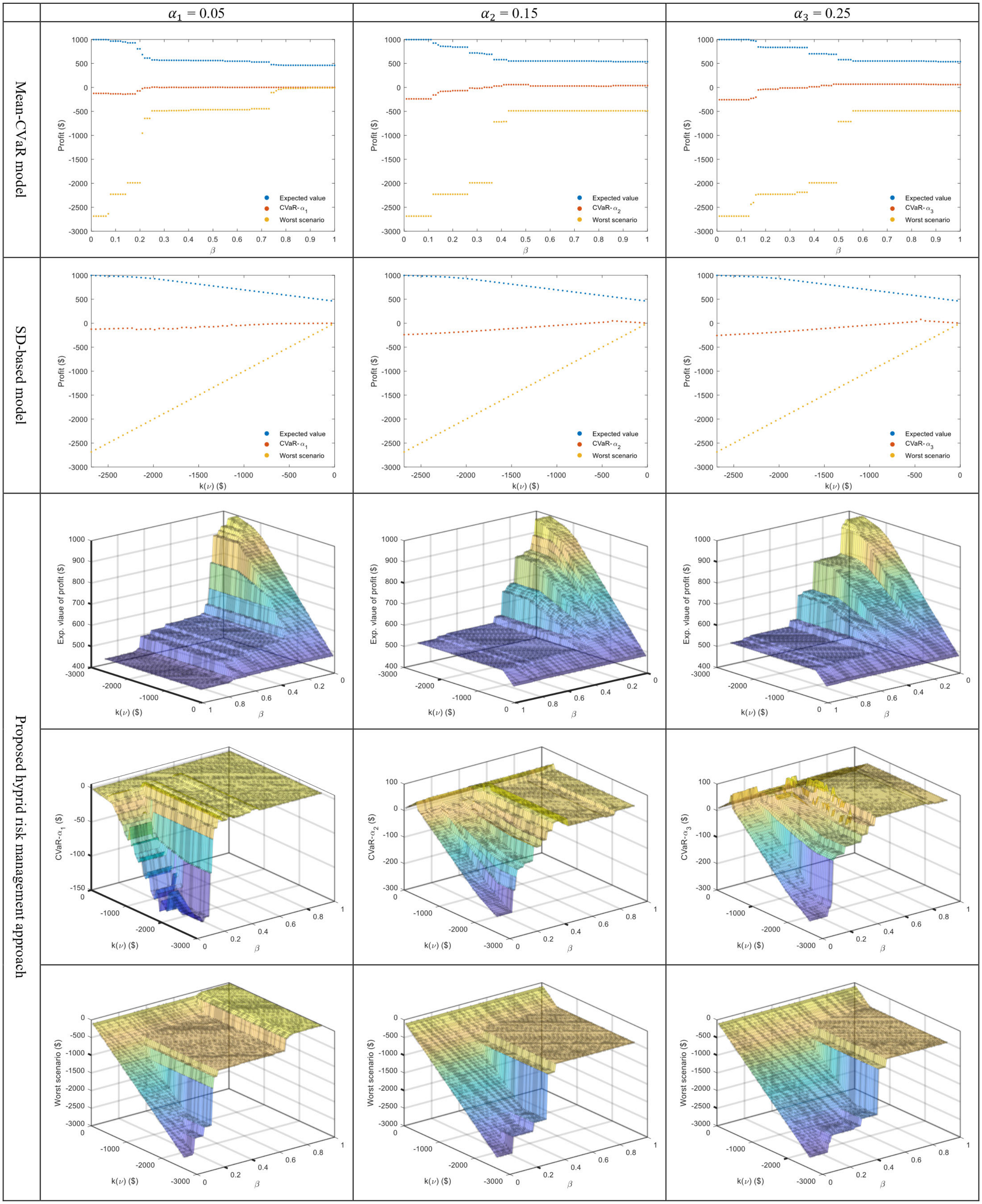

Fig. 4. The effect of different risk management parameters α, β, and $k(1)$ on the expected value, CVaR−α value, and worst scenario's value of the optimal objective's distributions obtained from the mean-CVaR model, the SD-based model, and the proposed hybrid risk management approach for a typical hour.