Second-Order Stochastic Dominance Constraints for Risk Management of a Wind Power Producer’s Optimal Bidding Strategy

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Abstract—Risk management is critical for wind producers to participate in electricity markets. Beside market price volatility and uncertainty, wind producers are facing an additional uncertainty in the level of wind power generation. Instead of using common risk measures, such as conditional value at risk (CVaR), this paper proposes the use of the second-order stochastic dominance constraints (SOSDCs) for risk management of wind producer’s bidding strategies. As benchmark selection is the major obstacle against applying SOSDCs, a novel optimization-based benchmark selection method is proposed. Case studies are carried out for an 80 MW wind producer using the SOSDCs-based bidding model with the proposed benchmark selection method and the CVaR-based bidding model. Results demonstrate the superior flexibility of the SOSDCs in risk management. Moreover, the SOSDCs can effectively manage the negative tail of the profit distribution. Compared to the SOSDCs, the CVaR is more suitable for modeling risk rather than managing risk, as it does not use a profit target value but uses the \((1 - \alpha)\)-quantile of the profit distribution. As the negative tail is the best representative of risk in the problem under study, the SOSDCs with the proposed benchmark selection method are more suitable than the CVaR for risk management of a wind power producer’s bidding strategy.

Index Terms—Bidding strategy, conditional value at risk (CVaR), electricity market, risk management, stochastic dominance, stochastic programming, wind energy.

I. INTRODUCTION

DEREGULATION in the electricity sector led to the creation of competitive electricity markets, where electricity is traded in the same way as other commodities. In a market environment, participants are exposed to financial risks due to uncertainty [1], where financial risk is defined as the possibility that a participant’s financial outcomes deviate adversely from what is expected [2]. In electricity markets, electricity prices are characterized by excessive volatility due to electricity’s special characteristics such as instantaneous delivery, limited storability, inelastic short-term demand, and compliance with Kirchhoff’s laws. Statistical data indicates that in the U.S., the...
average annual volatility of electricity price is 359.8%; while those of natural gas and petroleum, financial assets, metals, agriculture, and meat are just 48.5%, 37.8%, 21.8%, 49.1%, and 42.6%, respectively [1]. Hence, electricity market participants are facing a high price risk due to the high volatility of electricity price in the market. In the U.S., some market operators, such as Southwest Power Pool (SPP), allow wind power to participate in the electricity pool. In such a case, wind power producers must fulfill their commitments regardless any deviations in the real-time production caused by the uncertainty of wind energy, which is another factor causing financial risks to wind power producers in the electricity market.

A variety of studies have been carried out to mitigate the risk of bidding wind power in electricity markets caused by the uncertainty in wind energy. For example, combining wind energy with energy storage to cope with uncertainty has been studied [3]–[7], but may not be a cost-effective solution [8]. Combining wind and thermal energies in one bidding strategy to transfer risk from wind to thermal was discussed in [9]. In [10]–[13], stochastic programming was used to generate optimal bidding strategies for wind power producers to hedge against uncertainties when participating in the day-ahead or adjustment market. The stochastic programming problem is commonly formulated to maximize/minimize the expected value of the objective function’s distribution (or portfolio). However, this approach does not ensure that the impact of unacceptable scenarios in the probability distribution of the optimal objective function is mitigated.

Financial risk management based on financial theories can be a solution to hedge against these unacceptable scenarios of the optimal objective function.

Financial risk management can be defined as a procedure of shaping the optimal objective function’s distribution. Most, if not all, existing works in the literature managed the risks of bidding strategies using common risk measures such as variance [14], value at risk (VaR) [15], and conditional value at risk (CVaR) [2], [11], [16]–[18]. Variance does not distinguish between positive and negative deviations from the expected value. Hence, it is not compatible with the definition of risk in this paper, which focuses only on negative deviations. VaR is a widely used risk measure but does not fulfill the subadditivity axiom. Therefore, it is not a coherent risk measure. On the other hand, CVaR is a coherent risk measure with preferable mathematical characteristics in optimization [19] and, therefore, is most commonly used in electricity market applications. Stochastic dominance, rather than a risk measure, is a mathematical approach used in financial risk management [20], [21]. In that approach, stochastic dominance constraints were added to the set of constraints of the problem to force the optimal distribution of the objective function to outperform a predefined benchmark distribution (or simply called benchmark), which was selected and accepted by the risk manager. Using stochastic dominance constraints provides more flexibility for the risk manager to obtain an optimal portfolio (or objective function distribution) based on the risk preferences, which may be vital in some applications. However, compared to risk measures, it is not an easy task to select an appropriate benchmark for stochastic dominance constraints to ensure that the resulting decision-making model is feasible.

In the literature, limited research has been done on the use of stochastic dominance constraints for the risk management in power system planning and operation or electricity market applications. To the best of the authors’ knowledge, stochastic dominance constraints have been used in the work to determine an electricity retailer’s optimal participation in forward and short-term markets to meet its demands [22], [23], the optimal design and operation of a power system with distributed generation with uncertainties [24], [25], the optimal generation capacity expansion with uncertainty [26], the optimal portfolios for electric utility companies [27], [28], the optimal trading strategy for a virtual power plant (a cluster of diverse distributed energy resources) in bilateral contracts and electricity markets, the optimal self-scheduling of a large consumer considering market uncertainty [29], and the optimal bidding strategy for a wind power producer in the day-ahead market [30]. However, none of the existing work discussed how the benchmarks were selected, which is a major obstacle to implementing stochastic dominance constraints in risk management.

Motivated by the authors’ preliminary study in [30], this paper proposes the use of the second-order stochastic dominance constraints (SOSDCs) for the risk management of a wind power producer’s bidding model. The wind power producer participates in the day-ahead and balancing (real-time) markets and faces three statistically independent uncertainties, which are wind power generation, day-ahead clearing price, and real-time clearing price. The uncertainties are represented by scenarios in the stochastic-programming-based bidding model. The main contributions of this paper include the following:

1) Developed a stochastic bidding model using the SOSDCs for the risk management to generate the optimal bidding strategy for a wind power producer.

2) Proposed a novel optimization-based benchmark selection method to fulfill the risk manager’s preferences and ensure the feasibility of the bidding model. The proposed method is applicable not only to the bidding problem under study but to any stochastic programming problem with SOSDCs.

3) Conducted a comparative study between the CVaR and SOSDCs for managing the risks of a wind power producer’s bidding model to demonstrate the superior performance and more flexibility of the SOSDCs over the CVaR in managing the negative tail of the profit distribution.

The rest of this paper is organized as follows. Section II presents the market framework and the risk-neutral bidding model for a wind power producer, discusses different risk measures and risk management strategies, and presents the bidding model using CVaR to manage the risk. Section III presents the proposed bidding model for a wind power producer using the SOSDCs for the risk management and proposes an optimization-based benchmark selection method for the SOSDCs. Case studies for an 80 MW wind farm are carried out in Section IV to evaluate and compare the bidding models using CVaR and SOSDCs for the risk management. Section V summarizes the paper by concluding remarks.
II. MARKET FRAMEWORK AND TRADITIONAL BIDDING MODELS FOR A WIND POWER PRODUCER

A. Electricity Market Framework (Pool-Based)

A pool-based electricity market consisting of a day-ahead market and a balancing market is considered. The clearing sequence of the electricity market is shown in Fig. 1 [31]. The day-ahead market of Day d closes at 10:00 a.m. on Day d−1. The wind producers have to perform 14–38 hours ahead forecasts of their production from 00:00 to 24:00 of Day d to generate their hourly bidding strategies for Day d no later than 10:00 a.m. on Day d−1. The balancing market is cleared hourly on Day d to provide energy to cover both positive and negative generation deviations from commitment. For each hour, producers are paid for the cleared energy volume at the day-ahead clearing price. Moreover, in the real-time market, producers are paid for positive energy deviations and will pay for negative energy deviations at the real-time price.

In such a market framework, the objective of a wind power producer is to maximize the expected profit from trading in the day-ahead and balancing markets while managing the risks caused by the uncertainties.

B. Risk-Neutral Bidding Model for Wind Power Producer

The bidding problem of a wind power producer is subjected to three statistically independent sources of uncertainties: 1) wind generation, 2) day-ahead market clearing price, and 3) balancing market clearing price. These uncertainties are modeled as stochastic processes by a set of scenarios, where each scenario has a value and a probability of occurrence, and the bidding problem is modeled using the stochastic programming approach. The methods of scenario generation in [31] and scenario reduction in [32] are used to generate scenarios and reduce the number of generated scenarios for each random variable that represents a source of uncertainty.

Equations (1)–(4) represent the mathematical model for a wind power producer’s risk-neutral bidding strategy. It aims to maximize the expected profit of the wind power producer that participates in the competitive day-ahead and balancing markets without managing the risk.

\[
\text{Maximize } \sum_{\omega=1}^{N_D} \sum_{t=1}^{N_T} \left[ \lambda_{t,\omega}^D W_{t,\omega}^D + \lambda_{t,\omega}^r \left( W_{t,\omega}^{ac} - W_{t,\omega}^D \right) \right] d_t \quad (1)
\]

Subject to:

\[
0 \leq W_{t,\omega}^D \leq W_{t,\omega}^{\text{max}}, \quad \forall t, \omega \quad (2)
\]

\[
\left( \lambda_{t,\omega}^D - \lambda_{t,\omega'}^D \right) \left( W_{t,\omega}^D - W_{t,\omega'}^D \right) \geq 0, \quad \forall t, \omega, \omega' \quad (3)
\]

\[
W_{t,\omega}^D = W_{t,\omega'}^{D'}, \quad \forall t, \omega, \omega' : \lambda_{t,\omega}^D = \lambda_{t,\omega'}^D \quad (4)
\]

where the expected profit in the objective function (1) is equal to the revenue from the day-ahead market plus the revenue from the balancing market minus the cost for negative energy deviations in the balancing market. Constraint (2) limits the wind energy capacity to be traded in the day-ahead market to the maximum available installed capacity of the wind power producer. Constraint (3) assures a nondecreasing bidding curve. Constraint (4) constitutes the nonanticipativity conditions of the first stage decisions in the day-ahead market.

C. Risk Management

In stochastic programming problems involving random variables, it is usual to optimize the expected values of the objective functions [31]. The main disadvantage of this approach is the ignorance of other important features describing the objective function’s distribution, such as maximum, minimum, etc. To overcome this disadvantage, risk management should be included in the stochastic programming models to ensure that the risk of the selected objective function distribution does not exceed a certain limit.

The most common way to apply risk management in stochastic programming is to include a risk measure (or risk functional) in the problem formulation, as in the mean-risk approach (also called Markowitz approach) discussed in [33], [34]. Commonly used risk measures include variance, shortfall probability, expected shortfall, VaR, and CVaR. Variance penalizes all of the scenarios with the values different from the expected value even if they are higher than the expected value. Shortfall probability (i.e., the probability of the scenarios beyond a prefixed value) overcomes this disadvantage by penalizing the scenarios beyond a prefixed value only, but cannot detect/manage the shape of the objective’s distribution beyond this prefixed value, as shown in Fig. 2. Expected shortfall (i.e., the expected value of the scenarios beyond a prefixed value) overcomes this drawback by considering the expectation of the tail of the objective’s
distribution, but is not a coherent risk measure due to the use of a prefixed value. VaR solves the problem of using a prefixed value by replacing it with a decision variable in the optimization problem. However, it cannot detect the tail shape of the objective’s distribution as what the shortfall probability does. Moreover, VaR is not subadditive, and so not coherent. CVaR is the most commonly used risk measure in electricity market applications due to its major mathematical characteristics and performance features discussed in [19]. It can be expressed using a linear formulation, without the need for binary variables. Moreover, it is able to quantify the tail shape and is a coherent risk measure. For a given $\alpha$ in the range of $[0\%, 100\%]$, the CVaR is equal to the expected value of the scenarios with the profits smaller than the $(1-\alpha)$-quantile of the profit distribution.

D. Bidding Model Using CVaR for Risk Management

The risk management using CVaR can be implemented in the risk-neutral bidding model (1)–(4) of the wind power producer, and the resulting bidding model is expressed by (5)–(10).

$$\text{Max } W^D_{\omega, s}, \eta, s_\omega \left( 1 - \beta \right) \left( \sum_{\omega=1}^{N_\omega} p_{\omega, \omega} \sum_{t=1}^{N_T} \left[ \lambda^D_{\omega, t} W^D_{\omega, t} \right. \right.$$ 

$$+ \lambda^r_{\omega} \left( W^{ac}_{\omega, t} - W^D_{\omega, t} \right) \left. \right] d_t \right)$$ 

$$+ \beta \left( \eta - \frac{1}{1-\alpha} \sum_{\omega=1}^{N_\omega} p_{\omega, \omega} s_\omega \right)$$ 

Subject to:

$$0 \leq W^D_{\omega, t} \leq W^{max}, \quad \forall t, \omega$$ 

$$\left( \lambda^D_{\omega, t} - \lambda^D_{\omega, t'} \right) \left( W^D_{\omega, t} - W^D_{\omega, t'} \right) \geq 0, \quad \forall t, \omega, t'$$ 

$$W^D_{\omega, t} = W^D_{\omega, t'}, \quad \forall t, \omega, t' : \lambda^D_{\omega, t} = \lambda^D_{\omega, t'}$$ 

$$\eta - \left( \sum_{t=1}^{N_T} \left[ \lambda^D_{\omega, t} W^D_{\omega, t} + \lambda^r_{\omega} \left( W^{ac}_{\omega, t} - W^D_{\omega, t} \right) \right] d_t \right)$$ 

$$\leq s_\omega, \quad \forall \omega$$ 

$$s_\omega \geq 0, \quad \forall \omega$$ 

where $CVaR \left( \eta - \frac{1}{1-\alpha} \sum_{\omega=1}^{N_\omega} p_{\omega, \omega} s_\omega \right)$ is added to the objective function (5) to manage the risk; the risk aversion parameter $\beta$, ranging from 0 to 1, represents the risk manager’s appetite to take risk and controls the tradeoff between risk and expected profit; and the constraints (9) and (10) are added to linearize the CVaR term in the objective function (5) [35].

III. STOCHASTIC-DOMINANCE-BASED RISK MANAGEMENT

Although the CDF of a random variable provides complete information about its distribution, it may be too complicated to use it for risk management. That is why simple risk measures are commonly used to measure the risk levels of random variables. Recently, the stochastic dominance concept was proposed for risk management by adding stochastic dominance constraints to the set of constraints of a stochastic programming problem. The constraints impose a benchmark distribution that changes the feasible region of the optimization problem [20], [21]. All undesirable solutions are excluded from the modified feasible region, and the optimal portfolio obtained by solving the optimization problem will outperform the imposed benchmark defined according to the risk manager’s preference. Stochastic dominance constraints can be constructed in different orders; while the most commonly used are the first and second orders. The first-order stochastic dominance constraint makes the optimization problem non-convex; while the problem with the SOSDCs is convex. In both cases, a benchmark should be chosen carefully to avoid infeasibility of the problem.

To the best of the authors’ knowledge, the benchmarks used in the stochastic-dominance-based risk management models in the literature were usually selected heuristically; no work has presented a benchmark selection method or provided guidelines on how to select the benchmark. This obstacle is resolved in this paper by a novel optimization-based benchmark selection method that is applicable to any stochastic programming problem with SOSDCs.

A. Bidding Model Using SOSDCs for Risk Management

By adding the SOSDCs (15)–(17) to the risk-neutral bidding model (1)–(4), a bidding model with the SOSDC-based risk management is obtained as (11)–(17).

$$\text{Max } W^D_{\omega, s}, \eta, s_\omega \sum_{\omega=1}^{N_\omega} p_{\omega, \omega} \sum_{t=1}^{N_T} \left[ \lambda^D_{\omega, t} W^D_{\omega, t} + \lambda^r_{\omega} \left( W^{ac}_{\omega, t} - W^D_{\omega, t} \right) \right] d_t$$ 

Subject to:

$$0 \leq W^D_{\omega, t} \leq W^{max}, \quad \forall t, \omega$$ 

$$\left( \lambda^D_{\omega, t} - \lambda^D_{\omega, t'} \right) \left( W^D_{\omega, t} - W^D_{\omega, t'} \right) \geq 0, \quad \forall t, \omega, t'$$ 

$$W^D_{\omega, t} = W^D_{\omega, t'}, \quad \forall t, \omega, t' : \lambda^D_{\omega, t} = \lambda^D_{\omega, t'}$$ 

$$\eta - \left( \sum_{t=1}^{N_T} \left[ \lambda^D_{\omega, t} W^D_{\omega, t} + \lambda^r_{\omega} \left( W^{ac}_{\omega, t} - W^D_{\omega, t} \right) \right] d_t \right)$$ 

$$\leq s_\omega, \quad \forall \omega$$ 

$$s_\omega \geq 0, \quad \forall \omega$$ 

$$k_v - \left( \sum_{t=1}^{N_T} \left[ \lambda^D_{\omega, t} W^D_{\omega, t} + \lambda^r_{\omega} \left( W^{ac}_{\omega, t} - W^D_{\omega, t} \right) \right] d_t \right)$$ 

$$\leq s_{\omega, v}, \quad \forall \omega, v$$ 

$$\sum_{\omega=1}^{N_\omega} p_{\omega, \omega} s_{\omega, v} \leq \sum_{v=1}^{N_v} r_{\omega, v} \cdot \max \left( k_v - k_{v'}, 0 \right), \quad \forall v$$ 

$$s_{\omega, v} \geq 0, \quad \forall \omega, v$$ 

The benchmark is imposed in the model via the added SOSDCs (15)–(17). These constraints ensure that the optimal objective function’s distribution second-order stochastically dominates the predetermined benchmark distribution. The benchmark can have any number of scenarios $N_v$. Each scenario has a probability $r_{\omega, v}$ (or $r_{\omega, v'}$) and a prefixed value $k_v$ (or $k_{v'}$). The added SOSDCs and the imposed benchmark will change the...
bidding problem’s feasible region to exclude the solutions that exceed the risk limits defined by the risk manager. Hence, the optimal profit distribution obtained by solving the problem (11)–(17) will outperform (dominate) the predefined benchmark.

B. Proposed Benchmark Selection Method

Since the imposed benchmark changes the feasible region of the bidding problem, the bidding model (11)–(17) will be infeasible if the benchmark is not properly selected. To solve this problem, a novel method is proposed in this section to assist the risk manager in selecting the benchmark to fulfill the risk preference while keeping the bidding problem feasible. By solving the risk neutral problem (1)–(4), the optimal values of the decision variables in each scenario of the problem with a predetermined probability, the CDF of the optimal profit distribution of the risk neutral problem (1)–(4) can be obtained, as shown in Fig. 3 for a certain hour. Similarly, the CDF of the problem (5)–(10), which uses the CVaR with \( \beta = 1 \) and \( \alpha = 99\% \) to manage the risk, can be obtained for the same hour. Then, a yellow rectangular region in Fig. 3 is defined as follows. It ranges from 0 to 1 on the vertical axis. The left-hand-side border of the region is the value of the worst scenario of the optimal profit distribution of the risk neutral problem, which only maximizes the expected profit and totally ignores the risk. The right-hand-side border of the region is the value of the worst scenario of the optimal profit distribution obtained by solving the problem (5)–(10), which minimizes the risk regardless the expected profit. The left- and right-hand-side boarders of the region represent two extremes in the risk management. As any benchmark with a CDF lying in this region will ensure that the problem (11)–(17) remains feasible, the region is called the benchmark’s effective feasible region. Finally, the benchmark can be selected within the effective feasible region according to the number of scenarios and their probabilities determined by the risk manager’s preference.

A benchmark can have different numbers of scenarios. Table I lists the parameters and Fig. 4 shows the CDFs of three benchmarks with different numbers of scenarios, where the \( N_v \)-scenario benchmark is the general case. Any scenario \( \nu \ (\nu = 1, \ldots, N_v) \) has two parameters: probability \( \tau_\nu \) and prefixed value \( k_\nu \). The CDF of an \( N_v \)-scenario benchmark has a nondecreasing staircase shape within the benchmark’s effective feasible region. If part or all of the benchmark’s CDF is outside the effective feasible region, the optimization problem (11)–(17) may be infeasible. For instance, any one-scenario benchmark with a positive value for \( k_1 \) will make the problem infeasible. For the one-scenario benchmark, the prefixed value of the single scenario, \( k_1 \), defines the prefixed minimum profit limit \( X \). For a two-scenario benchmark, the first scenario can take any prefixed value \( k_1 = X \) within the benchmark’s effective feasible region, but the second scenario’s prefixed value is zero \( (k_2 = 0) \) to put a limit \( Y \) on the probability of the negative tail of the profit distribution.

The selection of the number of benchmark scenarios \( N_v \) and their probabilities \( \tau_\nu \) and prefixed values \( k_\nu \ (\nu = 1, \ldots, N_v) \) depends on the risk manager’s preference. A benchmark with more scenarios provides more flexible and, thus, better risk management. However, the computational cost of solving the problem (11)–(17) increases with the number of scenarios of the benchmark, because each scenario in the benchmark imposes \( 2N_\Omega + 1 \) constraints, where \( N_\Omega \) is the number of scenarios of the stochastic programming problem. As a tradeoff between risk management flexibility and computational cost, a benchmark with 1–3 scenarios would be enough for the wind power bidding.
problem studied in this work. As demonstrated by the case studies in Section IV, the risk management performance of the SOSDC-based bidding model using benchmarks with 1–3 scenarios outperforms that of the mean-CVaR model.

IV. Case Study Validation

Case studies are carried out for a wind farm in Nebraska, United States using the SOSDC-based bidding model (11)–(17) with the proposed optimization-based benchmark selection method. The results are compared with those of the CVaR-based bidding model (5)–(10) to show the advantages of using the SOSDCs for the risk management of the wind power producer’s bidding strategy in the short-term electricity market.

A. Simulation Setup

The wind farm has a total installed capacity of 80 MW. The uncertain variables of the problem include wind power generation, day-ahead price, and real-time price. They are modeled as statistically independent discrete random processes using the seasonal autoregressive integrated moving average (ARIMA) model. First, the seasonal ARIMA model [31] is applied to generate 500 scenarios by considering daily seasonality for each uncertain variable using the historical data of the variable obtained from the Southwest Power Pool (SPP). Then, the forward-selection-based scenario reduction technique [32] is applied to reduce the scenario numbers of the wind power generation, day-ahead price, and real-time price to 5. Therefore, totally 125 scenarios are generated for the bidding models. The bidding models of the wind producer are coded in MATLAB and solved using Gurobi Optimizer on a Windows desktop computer with a 3.2 GHz Core i5 CPU and 3 GB RAM. To obtain the day-ahead bidding curves for the wind power producer, the bidding models are solved hourly for the 24 hours of the next day. The execution times of the risk-neutral and mean-CVaR models for the cases studied are approximately 3.1 and 3.3 seconds, respectively. Meanwhile, the execution times of the SOSDC-based bidding model with one-, two-, and four-scenario benchmarks are approximately 4.1, 4.5, and 5.3 seconds, respectively. All of these execution times are acceptable for a bidding problem running on an hourly basis.

B. Benchmark’s Effective Feasible Region of the SOSDC-Based Bidding Model

The proposed benchmark selection method provides a general and systematic approach to ensure the feasibility of the bidding model. It should be mentioned that the benchmark’s effective feasible region illustrated in Fig. 3 or 4 is not fixed for different hours but depends on the values of the scenarios of the problem’s input random variables. The wind power bidding problem has three input random variables, among which the day-ahead and real-time market clearing prices can be positive or negative while the wind power production is always nonnegative. To illustrate how positive/negative values of the random variables affect the effective feasible region, four different cases listed in Table II are considered and the corresponding effective feasible regions are compared in Fig. 5. Both the cases A and D have a rectangular effective feasible region that is bounded by the worst scenarios of the two extreme cases of risk management, i.e., the risk-neutral and the most risk-averse settings. However, the effective feasible region of Case B or C is a vertical line, which indicates that the worst scenarios of the two extreme cases of risk management are equivalent. This happens because the price in one market is always higher than the price in the other market. For example, in Case B, the real-time price is always higher than the day-ahead price. Hence, regardless of the risk-aversion level, the rational decision is to bid all wind generation in the market with the higher price.

The scenarios of market prices usually have nonnegative values. Hence, without loss of generality, only nonnegative values are considered for the scenarios of market prices in the following discussions.

C. Bidding Model Using SOSDCs for Risk Management

Basically, a one-scenario benchmark is a vertical line, which forces the profit distribution obtained from the bidding model not to exceed its prefixed value, as illustrated in Fig. 6 for three different one-scenario benchmarks (dotted lines) used for the same hour. Each benchmark and the corresponding profit distribution in solid line are in the same color. Clearly, the worst profit scenario cannot exceed the prefixed value of the benchmark in each case. Although most of the aforementioned
Fig. 6. CDFs of a one-hour profit obtained from the SOSDC-based bidding model using three different one-scenario benchmarks. The yellow rectangle defines the benchmark’s effective feasible region.

Fig. 7. CDFs of a one-hour profit obtained from the SOSDC-based bidding model using three different two-scenario benchmarks. The yellow rectangle defines the benchmark’s effective feasible region.

D. Comparison of CVaR and SOSDCs for Risk Management

If the mean-CVaR approach is used to manage risk, the objective function is formulated to maximize the expected profit while minimizing the risk defined by the expectation of the predefined \((1 - \alpha)\)-quantile tail of the profit distribution, where \(\alpha\) ranges from 0% to 100%. The trade-off between the maximization and minimization is controlled by the risk-aversion parameter \(\beta\), which ranges from 0 to 1. On the other hand, if the SOSDCs are used for risk management, they impose a predefined benchmark to modify the problem’s feasible region. The benchmark can be imposed to flexibly modify the problem’s feasible region by changing its corner points. In this way, any point in the problem’s feasible region can be chosen to be the best corner point (the optimal solution) in the modified feasible region of the problem. Such flexibility is not achievable by managing the values of the risk management parameters \(\alpha\) and \(\beta\) in the mean-CVaR approach. Risk management can be defined as a procedure for shaping a portfolio distribution. Thus, the superior flexibility of the SOSDC approach, over the mean-CVaR approach, in selecting the optimal distribution of the objective function, makes it more suitable for the risk management of the bidding problem. This superior flexibility of the SOSDC approach can be demonstrated via comparing the results of the two approaches for the bidding problem with different risk management preferences that span the feasible ranges of the risk management parameters. First, the mean-CVaR with different combinations of \(\alpha = [0\%:1\%:99\%]\) and \(\beta = [0:0.01:1]\) are used in the bidding model (5)–(10) to manage the risk of the one-hour profit distribution of the wind power producer. Out of the 10,100 different cases tested, only 439 different optimal solutions are obtained and plotted in Fig. 9, indicating that many cases with different \(\alpha\) and \(\beta\) values have the same optimal solution. A single CDF of the optimal profit obtained from the SOSDC-based bidding model is also plotted in Fig. 9 and obviously cannot be represented by any of the 439 CDFs of the optimal profit.
Fig. 9. CDFs of a one-hour profit obtained from the CVaR-based bidding model using different combinations of $\alpha$ and $\beta$, and a CDF obtained from the SOSDC-based bidding model for the same hour.

Fig. 10. CDFs of a one-hour profit obtained from the SOSDC-based bidding model with one-scenario benchmark for different values of $X = [-2685:1:0]$. Obtained from the mean-CVaR-based bidding model. Then, the SOSDC-based bidding model (11)–(17), with a one-scenario benchmark and different values of $X = [-2685:1:0]$, is used to obtain the optimal profit distributions of the wind power producer for the same hour. The resulting CDFs of the optimal profit are plotted in Fig. 10, where 2686 different optimal solutions are obtained from the 2686 cases tested. Fig. 11 shows the effect of the prefixed value $X$ of the one-scenario benchmark on the expected value of the optimal profit distribution obtained from the SOSDC-based bidding model. As expected, an increase in the risk aversion level (i.e., the value of $X$) leads to a reduction in the expected profit. Although the same trend is observed in the result of the mean-CVaR bidding model, the SOSDC-based model can reach the expected profits that cannot be reached by the mean-CVaR model because the orange dots are covered by the blue dots.

The results in Figs. 9, 10, and 11 show that the SOSDC-based bidding model can offer more optimal solutions than the mean-CVaR-based bidding model even with less number of cases tested. In other words, the SOSDC-based bidding model can offer optimal solutions that cannot be offered by the mean-CVaR-based bidding model. Similar results were obtained for other hours of the bidding problem under study. However, it is important to mention that the mean-CVaR and SOSDC-based models would provide identical results at each of the two extreme cases of risk management, i.e., the risk-neutral and the most risk-averse settings.

In the wind power bidding problem under study, only the financial gain/loss from the electricity market participation is considered in the objective function; while the unit generation cost is ignored because it is either zero or constant through a power purchase agreement and does not depend on the market. In such a case, the scenarios with negative profit values (i.e., the negative tail) are considered as the risk. When these negative profit values and their probabilities are high, the portfolio’s risk is high.

The mean-CVaR approach, which manages the $(1 - \alpha)$-quantile tail, cannot manage the negative tail directly, as shown in Fig. 12, which shows the CDFs of the optimal hourly profits for 24 hours of a day were obtained from the mean-CVaR-based bidding model with $\alpha = 95\%$ and $\beta = 0.2$. Many CDFs still have large negative tails, which means high risks, as the $(1 - \alpha)$-quantile tails depend on other parameters of the problem, such as the probabilities and values of the uncertain variables’ scenarios, which change from one hour to another. To solve this problem, different values of $\alpha$ and $\beta$ should be used for different hours to manage the negative tail instead of the $(1 - \alpha)$-quantile tail. On the contrary, the SOSDCs with a fixed benchmark can be applied for all hours to manage the negative tails directly regardless of other parameters of the problem. For the same 24 hours of Fig. 12, the bidding model using the SOSDCs with...
and a novel optimization-based benchmark selection method was proposed to overcome the main obstacle against using the SOSDCs for risk management. Case studies were carried out for an 80 MW wind farm using the proposed SOSDC-based bidding model and the CVaR-based bidding model as CVaR is the most commonly used risk measure in electricity market applications. The effects of different parameters of the CVaR and SOSDC approaches were studied. Compared to the CVaR approach that only uses two parameters $\alpha$ and $\beta$ to represent the risk preference, the proposed SOSDC-based risk management approach provided more flexibility in representing the risk preference of the decision maker via defining a benchmark distribution with more parameters and is more efficient in managing the negative tail of the profit distribution, which is the best representation of the risk for the bidding problem under study. As risk management is a procedure of shaping a portfolio distribution, the SOSDC approach could offer optimal profit distributions that could not be offered by the CVaR approach, as demonstrated in the case studies. Compared to the SOSDCs, the CVaR is more suitable for measuring risk rather than managing risk, as it does not use a profit target value but the $(1 - \alpha)$-quantile of the profit distribution.

### REFERENCES


