Numerical Methods for Optimal Control of Hybrid Electric Agricultural Tractors

Chao Jia, Wei Qiao, and Liyan Qu
Power and Energy Systems Laboratory
Department of Electrical and Computer Engineering
University of Nebraska-Lincoln
Lincoln, NE, 68588, USA
cjia@huskers.unl.edu; wqiao3@unl.edu; lqu2@unl.edu

Abstract— **Electrification of agricultural machinery offers substantial benefits and has drawn attentions from manufacturers, so different types of hybrid electric agricultural tractors (HEATs) have been conceptually designed. In HEATs, energy management strategies play an important role to improve the performance of fuel economy. The previous studies are dominated by rule-based strategies. This work proposes optimization-based energy management strategies for HEATs to minimize the fuel consumption in typical working cycles. Based on the optimal control principles, dynamic programming, indirect method, and direct method are used to solve the formulated optimal control problem (OCP) and obtain three different optimal energy management strategies. All the three strategies achieve good performance in solving the OCP. Moreover, compared with the typical rule-based benchmarking strategy, these optimal EMSs show significant improvements in fuel efficiency (up to 5%). At last, the advantages and disadvantages of each method are illustrated and discussed by numerical simulations.**

Keywords—optimal control, energy management strategy, numerical methods, hybrid electric agricultural tractor.

I. INTRODUCTION

Agricultural tractors (ATs) play a crucial role in the modern agricultural production to feed an ever-growing population worldwide. However, the use of diesel engines in ATs leads to a large amount of pollutant emissions and fuel consumption. Electrification of ATs is considered as a proactive strategy to relieve these issues, since it offers great potential to improve fuel efficiency and reduce exhaust emissions. Some other benefits of electrified ATs are controllability, reliability, and comfort improvements. Therefore, a variety of electrified AT prototypes have been developed in recent years and can be categorized as non-hybrid diesel electric, hybrid diesel electric, battery electric, and fuel cell electric ATs.

Toward the electrification of vehicle powertrain, a considerable research has been focused on the development of supervisory energy management strategies (EMSs) for hybrid electric powertrains. The core issue is the power split between different power sources to satisfy vehicle power demand, minimize fuel consumption, or prolong battery lifetime. Since electrification of ATs lags electrification of on-road vehicles for years, limited research has been conducted to develop EMSs for hybrid electric ATs (HEATs). The existing studies mainly focused on deterministic or fuzzy rule-based EMSs [1]-[5].

The main purpose of this work is to investigate powertrain modeling and develop optimal EMSs for HEATs. In contrast to the rule-based EMSs in the literatures, the energy management problem is formulated as a nonlinear constrained optimal control problem (OCP) in this paper. Three numerical methods, i.e., a deterministic dynamic programming (DP), an indirect method based on Pontryagin's Minimum Principle (PMP), and a direct method based on nonlinear programming (NLP) are investigated to solve the OCP. Compared with the power follower strategy, one traditional benchmark rule-based EMS, these optimal EMSs can reduce 3%-5% fuel consumption. Besides, pros and cons of the three methods are discussed and verified by the numerical simulations.

The rest of this paper is structured as follows. The powertrain and power demand models of HEATs are presented in Section II. The energy management problem is formulated as an OCP with the objective to optimize the fuel economy. Section III introduces three different numerical approaches to solve the OCP. Numerical simulation results are provided and discussed in Section IV. Section V summarizes the work.

II. MODELING AND PROBLEM FORMULATION

The series HEAT configuration is considered, and the basic architecture is shown in Fig. 1. The plant model and low level control are described in [5], which represents a general-purpose HEATs. Control-oriented models are used in the design and test of energy mangement strategies.

A. Genset Model

Since the genset is mechanically decoupled from the wheels, it can be operated along the optimal operating line (OOL) to achieve the maximum efficiency [6]. By combining the engine and generator efficiency data, the genset efficiency map and OOL are plotted in Fig. 2, and the detail of this figure is explained in [5]. Then, the optimal fuel consumption rate $\dot{m}_f$ can be fitted as a fourth-order polynomial function of the genset output power P_g

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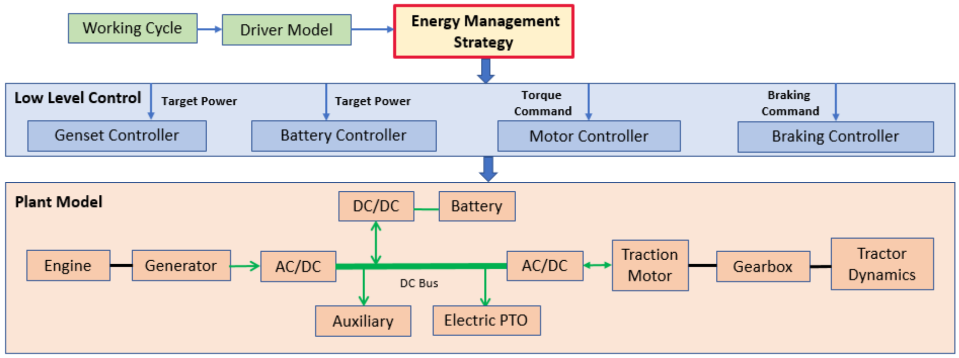

Fig. 1. Block diagram of the overall powertrain architecture of the series HEAT.

$$\dot{m_f}(P_g) = \sum_{0}^{3} a_i P_g^{3-i} \tag{1}$$

Fig. 3 illustrates the good accuracy of curve fitting results.

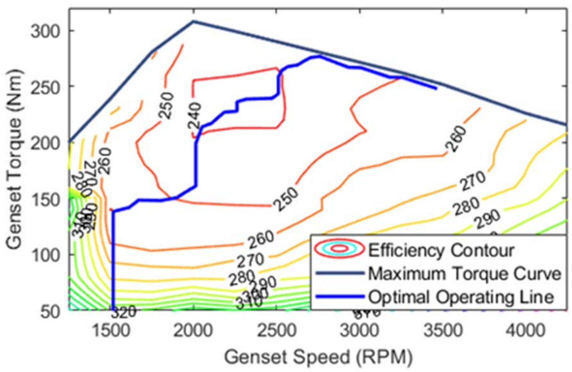

Fig. 2. Efficiency map and OOL of a genset.

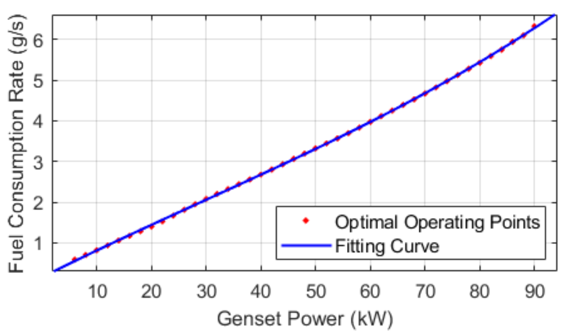

Fig. 3. Fuel consumption rate of the genset as a function of the output power.

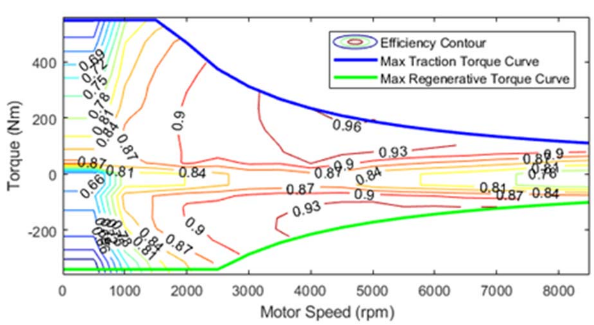

Fig. 4. Effeciency map of the combined motor and inverter.

B. Battery Model

The state of charge (SOC) of the battery is selected as the state variable, which is defined as

$$\dot{x}(t) = S\dot{O}C(t) = -\frac{I_b(t)}{Q_b} \tag{2}$$

where I_b is the battery current (positive during discharge) and Q_b is the nominal charge capacity [7]. By applying a typical internal resistance model to calculate the current, (2) becomes

$$\dot{x}(t) = -\frac{V_{oc}(x(t)) - \sqrt{V_{oc}^2(x(t)) - 4R_b(x(t))P_b(t)}}{2Q_b R_b(x(t))}$$

$$= f(x(t), P_b(t)) \tag{3}$$

where P_b is the battery output power, V_{oc} is the battery open circuit voltage, and R_b is the battery equivalent internal resistance.

C. Power Demand Model

According to vehicle longitudinal dynamics, the required tractive force at the driving wheels to propel a tractor-implement combination is determined by [8]:

$$F_t = \frac{1}{2}\rho_a C_d A_F v^2 + C_r mg\cos\theta + mg\sin\theta + \delta ma + F_{dr} \tag{4}$$

where F_t is the tractive force, ρ_a is the air density, C_d is the aerodynamic drag coefficient, A is the vehicle frontal area, v is the tractor velocity, C_r is the tire rolling resistance coefficient, m is the tractor mass, g is the gravity coefficient, θ is the road grade, δ is the equivalent inertia factor, a is the acceleration, and F_{dr} is the draft force for the implement. According to the standard developed by the American Society of Agricultural and Biological Engineers [9], the draft requirements can be calculated as

$$F_{dr} = F_i(A + Bv + Cv^2)WT \tag{5}$$

where F_i is a dimensionless soil texture adjustment parameter; A, B, and C are machine-specific parameters; W is the machine width or number of rows or tools; and T is the tillage depth for major tools or equal to 1 for minor tillage tools and seeding implements.

The power demand on the DC bus can be directly computed as follows.

$$P_{dem}(t) = \begin{cases} F_t v/\eta_m \eta_t \eta_f + P_{ePTO} + P_{au}, & F_t \geq 0 \\ F_t v \cdot \eta_m \eta_t \eta_f + P_{ePTO} + P_{au}, & F_t < 0 \end{cases} \quad (6)$$

where η_m is the combined efficiency of traction motor and inverter, which is plotted in Fig. 4, η_t is the transmission efficiency, η_f is the final reduction efficiency, P_{ePTO} is the power of electric power-take-off, and P_{au} is the auxiliary power. Based on the power balance, the power demand is satisfied by the genset and the battery

$$P_{dem}(t) = P_b(t) + P_g(t) \Rightarrow P_g(t) = P_{dem}(t) - P_b(t) \quad (7)$$

Since $P_{dem}(t)$ is calculated as a *priori*, $\dot{m}_f\left(P_g(t)\right)$ can be further described as $\dot{m}_f\left(P_b(t)\right)$.

D. Optimal Energy Management Problem

By selecting $P_b(t)$ as the control variable, the objective of the optimal energy management problem is to find the optimal profile or trajectory of the genset power to minimize the total fuel consumption over an entire working cycle.

$$J = \int_{t_0}^{t_f} \dot{m}_f\left(u(t)\right) dt \quad (8)$$

The optimization problem is subject to the following constraints:

State equation:

$$\dot{x}(t) = f\left(x(t), u(t)\right)$$

Boundary condition:

$$x(t_0) = x_0, \; x\left(t_f\right) = x_f$$

State constraints:

$$x_{min} \leq x(t) \leq x_{max}$$

Control constraints:

$$P_{b.min}(t) \leq u(t) \leq P_{b.max},$$

$$P_{dem}(t) - P_{g.max} \leq u(t) \leq P_{dem}(t) - P_{g.min}$$

where $P_{b.min}$, $P_{b.max}$, $P_{g.min}$ and $P_{g.max}$ are the minimum and maximum power of the battery and genset, respectively.

III. Numerical methods

Three groups of numerical methods have been commonly used to solve OCPs: DP, indirect method, and direct method [10]. In this work, the optimal energy management problem (8) are approached by all the three numerical methods.

A. Dynamic Programming

In DP, the original continuous-time model (3) is first discretized as follows.

$$x_{k+1} = f_D(x_k, u_k) \qquad k = 0,1,\dots,N-1 \quad (9)$$

Then, the objective function yields

$$J = \sum_1^{N-1} \dot{m}_{f.D}(u_k) \quad (10)$$

Based on Bellman's Principle of Optimality, the DP algorithm calculates the cost-to-go function at each node in the discretized-time state space by proceeding backward in time (bottom-up fashion) [11]. Then, the energy management problem (8) can be expressed in the following recurrent form.

$$J^*_{N-k,N}(x_{N-k}) = \min_{u_{N-k}} \{ \dot{m}_{f.D}(u_{N-k}) + J^*_{N-(k-1),N}\left(f_D(x_{N-k}, u_{N-k})\right) \} \quad (11)$$

where $J^*_{N-k,N}$ is the minimum cost-to-go from the time step $N-k$ to N.

By discretizing the continuous-time model and the state and control domains, the OCP is broken down into simpler discrete-time subproblems. Solving these subproblems yields local optimal solutions to the OCP, from which the global optimal control trajectory can be obtained. The MATLAB implementation of the DP adopted in this work is introduced in [12].

B. Indirect Method

Indirect methods attempt to find the necessary conditions for optimality, which are described according to the PMP for the continuous time OCP in terms of the Hamiltonian

$$H\left(x(t), \lambda(t), u(t)\right) = \dot{m}_f\left(u(t)\right) + \lambda(t) \cdot f\left(x(t), u(t)\right) \quad (12)$$

where $t \in \left[t_0, t_f\right]$, $u(t) \in U$, and U is the admissible control set. Let $u^*(t)$ be an optimal control policy and $x^*(t)$ and $\lambda^*(t)$ the resulting state and costate trajectories, respectively. Then, the necessary conditions for $u^*(t)$ are

ODE model:

$$\dot{x}^*(t) = \frac{\partial H}{\partial \lambda}\left(x^*(t), u^*(t), \lambda^*(t)\right) \quad (13)$$

Costate equation:

$$\dot{\lambda}^*(t) = -\frac{\partial H}{\partial x}\left(x^*(t), u^*(t), \lambda^*(t)\right) \quad (14)$$

Minimum principle:

$$u^*(t) = \arg\min_u H\left(x^*(t), \lambda^*(t), u(t)\right) \quad (15)$$

Initial and terminal conditions:

$$x^*(t_0) = x_0, \;\; x^*\left(t_f\right) = x_f \quad (16)$$

Since the boundary conditions are given at both the start and the end of the time horizon, these necessary optimality conditions lead to a two-point boundary value problem (TPBVP) that can be solved by the shooting method described as follows:

Solving PMP with the Shooting Method

```
Choose a sufficiently small threshold ϵ > 0. Discrete the time
horizon [t_0, t_f] into N intervals.
repeat
    Guess λ(t_0) for initial costate. λ_k = λ(t_0). k ← 1
        repeat
            Solve (15), u*_k = arg min H(x*_k, λ*_k, u_k).
            Integrate (13) and (14), and compute x*_{k+1} and λ*_{k+1}.
            k ← k + 1
        until k = N
    Update λ(t_0)
until |x*_N − x(t_f)| < ϵ
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C. Direct Method

The key idea of a direct method is approximating the state and control of the OCP in some appropriate manner, and then the original OCP is transcribed into a NLP problem [13]. This problem is solved by using powerful NLP solves. Typically, the direct methods approximate all continuous functions as polynomial splines.

The Legendre-Gauss-Radau (LGR) orthogonal collocation method is employed for discretization. The time interval $t \in [t_0, t_f]$ is transformed to a fixed interval $\tau \in [-1, +1]$, which is further divided into K mesh intervals $S_k = [T_{k-1}, T_k]$, $k = 1, \dots, K$. Next, the state and control variables are approximated with a basis of Lagrange polynomials [14].

$$x^{(k)}(\tau) \approx X^{(k)}(\tau) = \sum_{j=1}^{N_k+1} X_j^{(k)} L_j^{(k)}(\tau),$$

$$L_j^{(k)}(\tau) = \prod_{\substack{l=1 \\ l \neq j}}^{N_k+1} \frac{\tau - \tau_l^{(k)}}{\tau_j^{(k)} - \tau_l^{(k)}} \tag{17}$$

$$u^{(k)}(\tau) \approx U^{(k)}(\tau) = \sum_{j=1}^{N_k+1} X_j^{(k)} L_j^{*(k)}(\tau),$$

$$L_j^{*(k)}(\tau) = \prod_{\substack{l=1 \\ l \neq j}}^{N_k+1} \frac{\tau - \tau_l^{(k)}}{\tau_j^{(k)} - \tau_l^{(k)}} \tag{18}$$

where $\left(\tau_1^{(k)}, \dots, \tau_{N_k}^{(k)}\right)$ are the LGR collocation points in $S_k = [T_{k-1}, T_k)$, and $\tau_{N_k+1}^{(k)} = T_k$ is a noncollocated point. Finally, the energy management problem can be expressed as the following NLP.

$$J \approx \sum_{k=1}^{K} \sum_{j=1}^{N_k} \omega_j^{(k)} \dot{m}_f\left(U_j^{(k)}, \tau_j^{(k)}\right) \tag{19}$$

Defect conditions:

$$x_{i+1}^{(k)} - x_1^{(k)} - \sum_{j=1}^{N_k} I_{ij}^{(k)} f\left(X_j^{(k)}, U_j^{(k)}, \tau_j^{(k)}\right) = 0 \tag{20}$$

Initial and terminal conditions:

$$X_1^{(1)}(t_0) = x(t_0), X_{N_k}^{(k)}(t_f) = x(t_f)$$

where $k = 1, \dots, K$; $i = 1, \dots, N_k$; $j = 1, \dots, N_k$; and $I_{ij}^{(k)}$ is an integration matrix. The continuity of state in the interior mesh point gives $X_{N_k}^{(k)} = X_1^{(k+1)}$ and this constraint can be eliminated by treating them as the same variable. A general-purpose software GPOPS-II is utilized to implement the orthogonal collocation method [15], and the solutions is obtained by the nonlinear programming solver IPOPT [16].

IV. NUMERICAL SIMULATIONS AND DISCUSSIONS

Three numerical methods are implemented in MATLAB to solve the optimal energy management problem. Some important parameters of EMS are listed in TABLE I. The simulations are executed on a desktop computer with the Intel Core i7-6700 CPU @3.4GHz, 16G RAM.

TABLE I. VALUES OF MODEL PARAMETERS

Parameter	Value	Parameter	Value
x_0	0.5	m (kg)	6000
x_f	0.5	C_d	0.64
x_{min}	0.3	C_r	0.10
x_{max}	0.7	A_F (m^2)	3.8
$P_{g.max}$ (kW)	90	η_t	0.97
$P_{g.min}$ (kW)	0	A	390
$P_{b.max}$ (kW)	110	B	19.0
$P_{b.min}$ (kW)	-50	C	0

A. Power Demand

Based on the power demand model (6), the corresponding request power for plough cycle and transport cycle are calculated offline and shown in Fig. 5 and Fig. 6, respectively. Regenerative braking offers substantial benefits such as improving fuel economy and reducing emissions, particularly in urban traffic situations that involve frequent accelerating and braking. However, in agricultural applications, energy recuperation is relatively low due to the low velocity, high friction of tires on farmland and additional draft force for implements. Fig. 5 shows that no energy could be recuperated during the plough cycle. However, transportation task would be an exceptional scenario that is more likely to have a good regeneration result, as illustrated in Fig. 6. Moreover, the power profile of transport is more complex owing to frequent deceleration and acceleration.

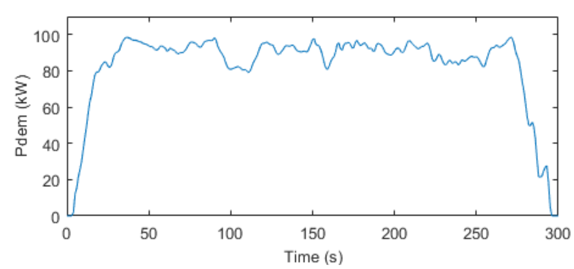

Fig. 5. Power demand of a plough cycle.

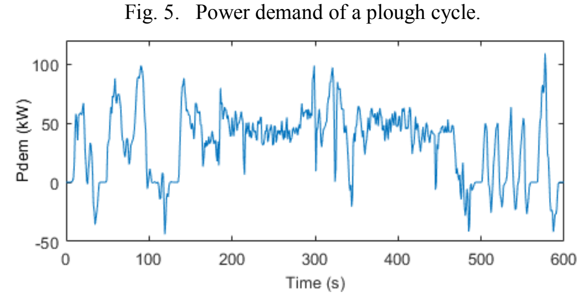

Fig. 6. Power demand of a transport cycle.

B. *SOC Trajectory*

Since the battery SOC is the state variable, it is insightful to compare SOC profiles for different numerical methods. Fig. 7 and 8 show the optimal SOC trajectories during the plough and transport cycles, respectively. Usually, DP is adopted to benchmark the performance of the EMS. The discretization grid of DP includes 80 state nodes and 320 control nodes. As illustrated in Fig. 7 and 8, the SOC trajectories obtained from the three methods matched with each other very well.

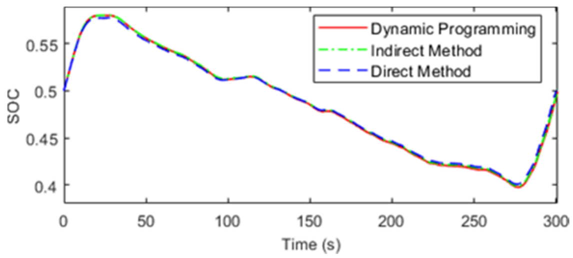

Fig. 7. Optimal SOC trajectory of a plough cycle

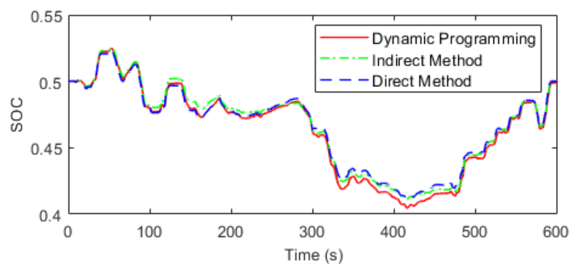

Fig. 8. Optimal SOC trajectory of a plough cycle

C. *Fuel Consumption*

Power follower control strategy is the most conventional rule-based EMS for series hybrid electric vehicles, and it is often adopted as the benchmark for the new proposed EMS. Therefore, the optimal EMSs designed in this paper are compared with power follower control in terms of the fuel consumption. Fuel economy simulation results of the different strategies for different working cycles are presented in Table II. It can be seen that optimal EMSs outperform the power follower strategy in the fuel economy performance.

TABLE II. FUEL CONSUMPTION RESULTS OF DIFFERENT STRATEGIES

Working Cycle	**Fuel Consumption [g]**				**Fuel Saving [%]**
	Dynamic Programming	**Indirect Method**	**Direct Method**	**Power Follower**	
Plough	1717.9	1718.5	1718.5	1772.6	3.08
Transport	1359.0	1359.1	1359.7	1431.5	4.99

D. *Pros and Cons*

Table III compares the computational time of the three methods, where DP is implemented with two different discretization grids: 80×320 and 160×640. It takes DP the least time in solving the problem while achieving good accuracy. It is also clear that the grid density affects the computational time significantly. While DP offers a theoretical way to solve multistage decision-making problems, it suffers from the inherent computational complexity, also known as the "*curse of dimensionality*," since the computational time grows exponentially with the number of state variables and inputs.

TABLE III. COMPUTATIONAL TIME OF the THREE METHODS.

Working Cycle	**Dynamic Programming**		**Indirect Method**	**Direct Method**
	80×320	160×640		
Plough	5.74s	17.88s	7.75s	7.71s
Transport	10.09s	33.05s	12.39s	16.23s

The optimality of the solution obtained from the indirect method is highly sensitive to the initial costate value λ_0. Fig. 9 shows the SOC trajectories obtained from the indirect method by using different initial values of the costate. As the absolute value of λ_0 increases, the SOC increases over the working cycle, and vice-versa. Furthermore, only one value of the costate can guarantee the charge-sustaining, and different cycles require different initial values. Some major pros and cons of the three methods are summarized in Table IV.

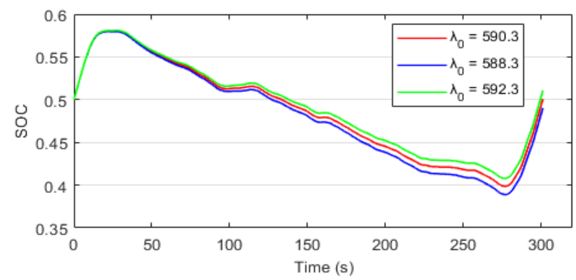

Fig. 9. Effect of initial costate values on the SOC trajectory obtained from the indirect method.

V. CONCLUSIONS

In recent years, there has been a clear trend towards electrification of agricultural equipment by transferring key technologies from on-road vehicles to ATs. Advanced energy management strategy has been a key factor to enable electrified tractors to compete with conventional tractors.

In contrast to the rule-based EMSs in the literatures, this work proposed optimization-based EMSs for the hybrid electric agricultural tractors. Energy management problem was formulated into a nonlinear constrained optimal control problem (OCP) to minimize the fuel consumption. Three different optimization methods, including a DP, a PMP-based indirect method, and an NLP-based direct method were used to solve the OCP. Simulation results demonstrated that all the strategies achieved similar performance in regulating the power between diesel engine and battery. Compared with the rule-based benchmark strategy, these optimal EMSs offer good improvements in fuel economy (up to 5%). Moreover, the advantages and drawbacks of the three methods were also discussed in this work.

The optimal EMSs in this work only optimize the fuel consumption in the working cycle. Future work may consider battery degradation in the optimization framework, which can prolong the battery lifetime.

TABLE IV: PROS AND CONS OF THE THREE METHODS.

	Dynamic Programming	Indirect Method	Direct Method
Pros	Global optimality is guaranteed.	Concept is simple. Control constraints are easy to deal with.	Powerful NLP solvers are available. Sophisticated discretization can be used.
Cons	Curse of dimensionality.	Good initial value of costate is crucial. Integration of the system may be unstable.	There are many variables and constraints in NLP. Approximate solutions may not converge.

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