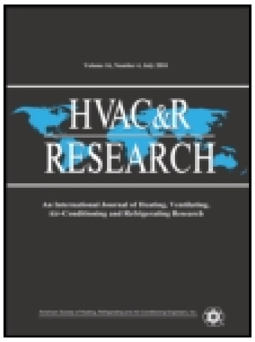

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A Generic Rating-Data-Based (GRDB) DX Coils Modeling Method

Huojun Yang **Haorong Li, PhD**
Member ASHRAE

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This paper presents an effective and easy to implement cooling modeling methodology for DX coils. The proposed model was created using publicly available HVAC manufacturer rating data and a reconstruction of a traditional cooling model. Specifically, a wet curve was constructed based on the manufacturer's rating data for the wet coil condition. Extrapolating a dry line from the critical point of this wet curve gave the dry coil conditions. Data from experimental tests of HVAC units are compared with the manufacturers' rating data to further validate the effectiveness of the model. Experimental results indicate that the method accurately predicts both wet-coil and dry-coil conditions (e.g., the relative error is below 9.82%, and the average absolute relative error is below 4.4%). It is hoped that this modeling methodology will not only increase the accessibility of packaged HVAC units to common users, but also better facilitate unit design, maintenance, and real-time automatic fault detection and diagnosis.

INTRODUCTION

Space cooling and refrigeration consumed a total of 3.0 quads of primary energy in US commercial buildings in 2006, and this consumption accounted for 17% of the total primary energy consumption in commercial buildings (D&L International Ltd. 2009). Katipamula and Brambley (2005) estimated that poorly maintained, degraded, and improperly controlled equipment wastes 15% to 30% of energy used in commercial buildings. Therefore, energy efficiency and conservation for the cooling and refrigeration sector becomes very important. How to model cooling coils (or cooling systems) will play an essential role in different aspects of cooling efficiency improvement, including optimal design, efficient operation, maintenance, and fault detection and diagnosis (FDD).

Traditional cooling coil analysis methods like those presented by Carrier et al. (1959) and ASHRAE (1993) are typically used in the common packaged HVAC cooling systems. The total cooling capacity ($\dot{Q}_T$), sensible cooling capacity ($\dot{Q}_S$), and sensible heat ratio (*SHR*) are determined by the apparatus dew point (ADP)/bypass factor (*BF*) approach, a method similar to the NTU-effectiveness calculations needed for the heat exchanger. However, there are several limitations with these methods. The first problem is that appropriate data are often difficult to obtain. The bypass factor, for example, is not always easily available. In fact, many cooling system manufacturers (e.g., manufacturer A, B, and D in Table 1) do not post the parameter in their ratings' manual or other related technical documents. As a result, users must acquire the missing data by performing calculations. Unfortunately, the procedure for these calculations is complicated, often requiring users to find the rated condition, performance curve, and dry-out point through iteration calculations or trial and error. Further complicating matters, the traditional methodology relies heavily on users' workmanship because

Huojun Yang is a PhD student and **Haorong Li** is an assistant professor in the Department of Architectural Engineering, University of Nebraska-Lincoln.

Table 1. Available Cooling System Data for Differing Manufacturers

Available Engineering Data		Manufacturers			
		A	**B**	**C**	**D**
General data	Unit model number	X	X	X	X
	Nominal tonnage	X	X	X	X
	Refrigerant charge	X	X	X	X
	Refrigerant type	X	X	X	X
	Expansion Device type	X	X	X	
	EER	X	X	X	X
	Cooling system's power	X	X		
	Electrical characteristics (voltage, hertz, phase no.)	X		X	X
Cooling system's capacity rating data	Entering dry-bulb temperature (evaporator), ET_{db}	X	X	X	
	Entering wet-bulb temperature (evaporator), ET_{wb}	X	X	X	X
	Return dry-bulb temperature, *RAT*				X
	Bypass factor, *BF*			X	
	Ambient temperature, *OAT*	X	X	X	X
	Airflow cfm (evaporator), *CFM*	X	X	X	X
	Stages' number, *n*	X			
	Total cooling capacity, $\dot{Q}_T$	X	X	X	X
	Sensible to total cooling ratio, *SHR*	X	X	X	X
	Compressor motor (total) power input, *Power*	X		X	X
Compressor	Compressor type	X	X	X	X
	Compressor quantity	X	X	X	X
Condenser coil	Net (total) face area	X	X	X	X
	Tube diameter	X	X		
	Number of rows	X	X	X	X
	Fins per inch	X		X	X
Condenser fan	Motor horsepower	X	X	X	X
	Motor rpm	X	X	X	
	Nominal air volume	X	X	X	X
	Diameter	X	X	X	X
	Number of blades	X			
Evaporator coil	Net (total) face area	X	X	X	X
	Tube diameter	X	X		
	Number of rows	X	X	X	X
	Fins per inch	X		X	X
Evaporator blower	Nominal motor output	X	X	X	X
	Motor rpm	X	X	X	X
	Total (nominal) air volume	X	X	X	X
	Wheel nominal diameter X width	X			X
	Blower type	X	X	X	X

the overall accuracy of the methodology is sensitive to the selection of the rated condition, and the performance is unacceptable under certain circumstances (Yang and Li 2009).

More specifically, in terms of the real-time online FDD in the real HVAC industry of commercial buildings, the traditional cooling coil models cannot be implemented in a simple, fast, and effective manner, and require a lot of calculation power. In this application, HVAC data are gathered by an FDD platform from remote buildings for every interval (e.g., one minute). Then the platform's server decides whether an HVAC has faults or energy saving potentials by analyzing these data. In addition, there are thousands of buildings, and in each building there are around 20–40 HVAC machines installed with different manufacturers and different machine types. Therefore, an alternative method combining the data-driven approach's characteristics (e.g., simple, effective, and accurate) with the free manufacturers' data (low cost) is proposed to support the platform's high speed and efficiency without sacrificing accuracy.

Although researchers have previously attempted to model AC components using the rating data from manufacturers, the method using manufacturers' data has not yet been applied to the operation of the entire cooling system. Li and Braun (2008), for example, employed manufacturers' data to study adjustable throat-area expansion valves, a single component of the HVAC system. Reichler (1999) developed a model for the cooling system from the manufacturers' data, but his research applied only to full-load and idealized working conditions. This paper, therefore, attempts to tackle the shortcomings of the current cooling coil's model methods with a new method, termed "GRDB," which employs manufacturers' cooling performance data to build a model viable under all operating conditions. We believe that the GRDB model can be implemented to facilitate automated on-line diagnosis and optimization of building energy systems and also be adopted to improve the cooling coil model in existing building energy simulation software (Yang and Li 2009).

MODELING MECHANICS

Analysis of Manufacturers' Data

As indicated in the introduction, the objective of this research is to develop an effective cooling model based on a generic modeling methodology and easily accessible ratings data. In order to build a model for analysis, published ratings data from the following four common cooling system manufacturers—A, B, C, and D—was organized in Table 1 according to the methods described as follows:

1. Manufacturers often use different names to identify the same variable. In order to minimize the confusion resulting from this practice, a single term was selected to represent each variable. For example, a device defined by manufacturer A as an "evaporator blower" is called an "indoor fan" by manufacturer B. In the table, "evaporator blower" was selected as a uniform term to identify this device for both manufacturers.
2. If unstated variables can be easily derived from knowledge of a stated variable, then those unstated variables are also counted as accessible information. For example, manufacturer D provides the values for the total cooling capacity and the sensible cooling capacity, but not the *SHR* value in its manual. The *SHR* value is still considered an available variable, however, since it is easily calculated from the equation, $SHR = \dot{Q}_S / \dot{Q}_T$.
3. Variables that have similar characteristics but are made from different manufacturers are categorized together. For example, "compressor motor power" of manufacturer A and C is categorized in the "total power input" category along with "compressor motor and condenser fan power" of manufacturer D.

To facilitate the modeling of the cooling system, variables affecting the cooling system's capacity are further divided into outputs—$\dot{Q}_T$, *SHR*, and *Power*—and inputs (all other remaining variables). A functional relationship between the outputs and inputs was formulated as shown in Table 2. In certain cases, variables that are intermediate between outputs and inputs are included to emphasize the introduction of the variable.

For example, manufacturer C uses the variable *BF*, a function of *CFM* and ET_{wb}, i.e., $BF = f(CFM, ET_{db})$, but Table 2 states the *BF* without variables *CFM* and ET_{wb}.

Challenges associated with combining Tables 1 and 2 with actual rating data from the manuals are listed below. Available rating data for the dry-coil condition are limited. The majority of the data presented in manuals are in the wet coil condition. (Entering air with an *SHR* equaling 1 is defined as being in the dry-coil condition, and data with an SHR < 1 is considered to be in the wet-coil condition). Therefore, HVAC system modeling using a traditional black box is not very effective, especially for extrapolation purposes in the dry-coil condition. In fact, manufacturer C clearly states in its manual: "Direct interpolation is permissible. Do not extrapolate" (Product Data).

1. Effective use of the ratings manual is hindered by the fact that manufacturers provide differing variables and a format that is not uniform. For example, while manufacturer C gives the *BF*, manufacturer D uses *RAT*, and manufacturers A and B give ET_{db} as an input value. Manufacturer A is the only one that uses *n* as an input. Table 2 shows that depending on the manufacturer, each output has a different input function. In addition, data of variables for some

Table 2. Relationship between the Inputs and Outputs of Manufacturers' Cooling Rating Data

Manufacturers	**Inputs**							**Outputs**
	ET_{wb}	ET_{db}	*BF*	*RAT*	*OAT*	*CFM*	*n*	$\dot{Q}_T$
A	X				X	X	X	$= f(ET_{wb}, OAT, CFM, n)$
B	X	X			X	X		$= f(ET_{wb}, ET_{db}, OAT, CFM)$
C	X		X		X	X		$= f(ET_{wb}, BF, OAT, CFM)$
D	X				X	X		$= f(ET_{wb}, OAT, CFM)$
								SHR
A	X	X			X	X	X	$= f(ET_{wb}, ET_{db}, OAT, CFM, n)$
B	X	X			X	X		$= f(ET_{wb}, ET_{db}, OAT, CFM)$
C	X		X		X	X		$= f(ET_{wb}, BF, OAT, CFM)$
D	X			X	X	X		$= f(ET_{wb}, RAT, OAT, CFM)$
								Power
A	X				X	X	X	$= f(ET_{wb}, OAT, CFM, n)$
B								
C	X		X		X	X		$= f(ET_{wb}, BF, OAT, CFM)$
D	X				X	X		$= f(ET_{wb}, OAT, CFM)$

HVAC product series are completely unlisted. Although *SHR* in manufacturer C is determined by the *BF*, the manufacturer does not define how to determine *BF* in product series such as the "50TFF*" and "50TJ*." Manufacturer B does not list the output as well.
2. The definition of the *Power* output differs among manufacturers. For example, manufacturer A and C refer to the term as a compressor motor power input, while manufacturer D defines *Power* as the total power input including the compressor motor and condenser-fan power.

The following solutions are proposed to lessen the confusion that may result from the above differences:

1. The underlying physical principle based on the equation $SHR = \dot{Q}_S/\dot{Q}_T$, as shown in Section 2.2, categorizes all data as either in the dry or wet coil condition, where the traditional black box model is substituted with a grey box so that interpolation and extrapolation are possible.
2. Manufacturers should adhere to a common terminology to lessen confusion. This can be accomplished by replacing variables with equivalent terms.

   BF of manufacturer C can be replaced by ET_{db} since $BF = f(CFM, ET_{db})$. *CFM* is an existing input, and the table for $BF = f(CFM, ET_{db})$ is usually given.

   RAT of manufacturer D can be substituted with ET_{db} following a replacement in the mathematical expression since $ET_{db} = f(\alpha, OAT, RAT)$, where α is the fixed fresh air ratio and *OAT* is an existing input.

   n of manufacturer A can be considered a known variable. The manuals of manufacturers other than manufacturer A do not display the variable, or consider *n* equal to 1. They automatically control the number of compressor stages according to the different inputs.

   After rearranging Table 2, the following equations are used to represent the cooling system's relationship between the outputs and inputs in manufacturers' data:

$$\dot{Q}_T = f(ET_{wb}, OAT, CFM) \tag{1}$$

$$\dot{Q}_T = f(ET_{wb}, ET_{db}, OAT, CFM) \tag{2}$$

$$SHR = f(ET_{wb}, ET_{db}, OAT, CFM) \tag{3}$$

$$\dot{Q}_S = \dot{Q}_T \cdot SHR \tag{4}$$

$$Power = f(ET_{wb}, OAT, CFM) \quad \text{or} \quad Power = f(ET_{wb}, ET_{db}, OAT, CFM) \tag{5}$$

3. We suggest that manufacturers present data that are complete and uniform.
4. The *Power* output is not analyzed in this paper due to a scarcity of relevant data.

Model Format

The cooling model developed in this paper is based upon Equations 1 (or 2), 3, and 4. These equations must satisfy the physical principle of a typical packaged HVAC cooling system such as that shown in Figure 1. In Figure 1, the four major components of the cooling system, including their outputs (e.g., heat rejection and cooling capacity), are displayed within the dashed line, and the cooling capacity is represented using $\dot{Q}_T$ and $\dot{Q}_S$. The area outside the dashed box for a designed, installed, and operational rooftop cooling system represents the system's input variables, including the *OAT* and the airflow rate at the condenser side (CFM_{Cond}); ET_{db}, ET_{wb}, and *CFM*; *n*, ν, and *I*. In this figure, the voltage and current (ν, *I*) are assumed to change insignificantly under

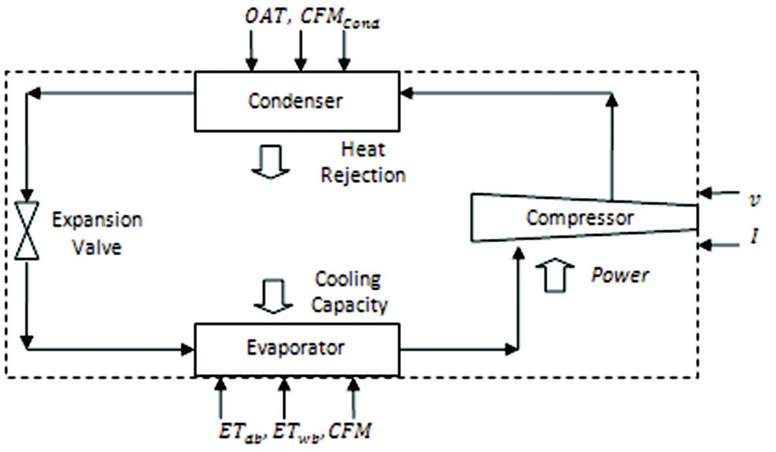

Figure 1. Cooling system for a packaged HVAC unit.

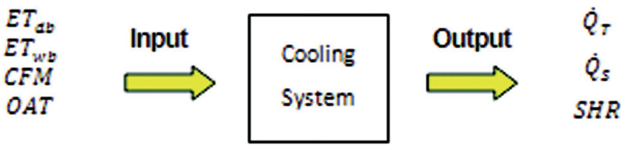

Figure 2. Relationship between inputs and outputs of a packaged HVAC cooling systems.

steady conditions. In addition, is also assumed to only slightly influence the system cooling capacity when the system runs at various operational conditions. Thus, the effects of (CFM_{Cond}, v, I) may be considered negligible. The inputs and outputs for an operational rooftop unit cooling system are shown in Figure 2.

In general, the set of equations about $\dot{Q}_T$, $\dot{Q}_S$, and SHR (i.e., Equations 2, 3, and 4) give the general function of the cooling system's outputs based on inputs from the physical model and match those (i.e., Equations 1, 2, 3, and 4) obtained from the ratings manual with the exception of $\dot{Q}_T$. The manual provides an additional expression (i.e., Equation 1) for this value such that $\dot{Q}_T = f(ET_{wb}, OAT, CFM)$. Selection of the appropriate equation—$\dot{Q}_T = f(ET_{wb}, ET_{db}, OAT, CFM)$, $\dot{Q}_T = f(ET_{wb}, OAT, CFM)$, and/or an alternative equation—is dependent on whether the entering air is in the dry-coil or wet-coil condition. This is discussed in greater detail in the following sections.

Wet-Coil Condition. Equations 6 and 7 are based on the driving conditions shown in Figure 3, where C_{wb-h} is the specific heat of wet coils and T_{evap} is the temperature of refrigerant entering evaporator.

$$\dot{Q}_T = CFM \cdot C_{wb-h} \cdot (ET_{wb} - T_{evap}) \tag{6}$$

$$T_{evap} = f(OAT, CFM, ET_{wb}) \tag{7}$$

C_{wb-h} is a constant for a specific operational evaporator; combining the two previous Equations 6 and 7 results in Equation 1.

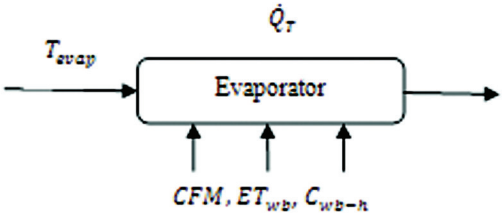

Figure 3. Driving conditions for the wet evaporator coil.

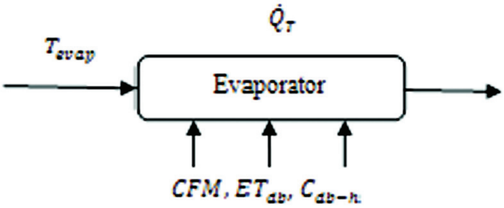

Figure 4. Driving conditions for the dry evaporator coil.

When analyzing $\dot{Q}_T$ for the wet-coil condition, the negligible effects of the ET_{wb} can also be explained by the factors listed below. The higher the entering air humidity ratio ω_{ET}, the greater the accuracy of the results.

1. ET_{wb} is determined by both ET_{db} and ω so that $ET_{wb} = f(ET_{db}, \omega_{ET})$. The effect of ET_{db} can be expressed by ET_{wb}.
2. ET_{wb} is an overall driving force of $\dot{Q}_T$ at the evaporator side for the sensible and latent loads.

Dry-Coil Condition. By analyzing the driving conditions shown in Figure 4, where C_{db-h} is the specific heat of dry coils, Equations 7 and 8 are obtained.

$$\dot{Q}_T = CFM \cdot C_{db-h} \cdot (ET_{db} - T_{evap}) \tag{8}$$

Also, C_{db-h} is a constant for an operational evaporator; combining the two previous Equations 7 and 8 gives $\dot{Q}_T = f(ET_{db}, CFM, OAT)$.

The effects of ET_{wb} can also be ignored when deriving the $\dot{Q}_T$ for the dry coil condition. This is because compared with ET_{wb}, the ET_{db} has a greater effect on the $\dot{Q}_T$. The latent heat, however, does not contribute to the $\dot{Q}_T$ value.

$$\text{Cooling model format}\begin{cases}\text{Wet-coil condition}\begin{cases}\dot{Q}_T = f(ET_{wb}CFM, OAT)\\ SHR = f(ET_{wb}, ET_{db}, OAT, CFM)(1 > SHR \geq 0)\\ \dot{Q}_S = SHR \cdot \dot{Q}_T\end{cases}\\ \text{Dry-coil condition}\begin{cases}\dot{Q}_T = f(ET_{db}, CFM, OAT)\\ SHR = 1\\ \dot{Q}_S = SHR \cdot \dot{Q}_T\end{cases}\end{cases} \tag{9}$$

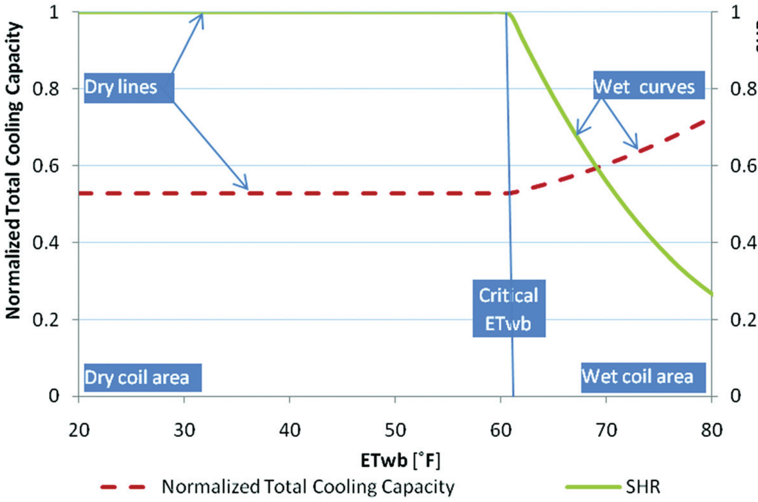

Figure 5. Relationship of $\dot{Q}_T$ and SHR with ET_{wb} at a fixed (ET_{db}, CFM, OAT).

Equation 9 shows a summary of the cooling model for the wet- and dry-coil conditions, and Figure 5 is a representation of the cooling model format. In Figure 5, the relationship of $\dot{Q}_T$ and SHR with ET_{wb} is indicated for an operational condition at fixed inputs (ET_{db}, CFM, OAT), where $\dot{Q}_T$ is normalized (i.e., the actual $\dot{Q}_T$/the rating capacity). The figure above shows two critical points: (ET^0_{wb}, $\dot{Q}^0_T$) and (ET^0_{wb}, $SHR = 1$), with a vertical line ($ET_{wb} = ET^0_{wb}$) passing through them. This line divides the graph into two sections: the left section represents the dry-coil condition (i.e., when the real $ET_{wb} < ET^0_{wb}$), and the right side the wet-coil condition (i.e., when the real $ET_{wb} > ET^0_{wb}$). In the dry coil section, both $\dot{Q}_T$ and SHR are constants ($SHR = 1$ and $\dot{Q}_T = \dot{Q}^0_T$) because the ET_{wb} has no effect on them at a fixed (ET_{db}, CFM, OAT). In the wet coil region, both $\dot{Q}_T$ and SHR will vary with an increasing ET_{wb} (SHR will decrease and $\dot{Q}_T$ will increase).

Critical Points. As shown in Figure 5, the SHR in wet-coil region varies between $1 > SHR \geq 0$) and increases with the decrease of ET_{wb}. There is also a critical point (ET^0_{wb}, $\dot{Q}^0_T$) when SHR is close to 1, where

$$\left.\begin{aligned} \lim_{SHR \to 1^-} ET_{wb} &= ET^0_{wb} \\ \lim_{SHR \to 1^-} \dot{Q}_T &= \dot{Q}^0_T \end{aligned}\right\}.$$

Furthermore, critical points (ET^0_{wb}, 1) and (ET^0_{wb}, $\dot{Q}^0_T$) separate curves ($SHR - ET_{wb}$) and ($\dot{Q}_T - ET_{wb}$) into wet curves and dry lines, respectively. There are continuous points in respective curves or lines.

$$\left.\begin{aligned} \lim_{ET_{wb} \to ET^{0-}_{wb}} SHR &= \lim_{ET_{wb} \to ET^{0+}_{wb}} SHR = 1 \\ \lim_{ET_{wb} \to ET^{0-}_{wb}} \dot{Q}_T &= \lim_{ET_{wb} \to ET^{0+}_{wb}} \dot{Q}_T = \dot{Q}^0_T \end{aligned}\right\}$$

Table 3. Rating Data Statistics for Cooling System Operation

Inputs	Values (I-P)	Values (SI)	Number of Different Values
CFM	5600, 7000, 8400 (ft^3/min)	2.64, 3.30, 3.96 (m^3/s)	3
OAT	85, 95, 105, 115 (°F)	29.4, 35, 40.6, 46.1 (°C)	4
ET_{wb}	63, 67, 71 (°F)	17.2, 19.4, 21.7 (°C)	3
ET_{db}	75, 80, 85 (°F)	23.9, 26.7, 29.4 (°C)	3

Summary of Cooling Format. The previous analysis is based on an ideal case, as cooling system disturbances or inaccuracies such as measurement errors make it difficult to determine an exact ET^0_{wb} value. The operating conditions within a close vicinity to the critical points on the graph are considered "mixed" (situated in both dry-coil and wet-coil conditions) and are influenced by both ET_{db} and ET_{wb} values. Thus, the relationship between the inputs and outputs in the mixed condition can be expressed in Equation 10.

$$\text{Mixed Condition}\begin{cases} \dot{Q}_T = f(ET_{db}, ET_{wb}, CFM, OAT) \\ SHR = f(ET_{wb}, ET_{db}, OAT, CFM), (0 \ll SHR \le 1) \\ \dot{Q}_S = SHR \cdot \dot{Q}_T \\ \left|ET_{wb} - ET^0_{wb}\right| \le \delta \text{ and } 0 < \delta \ll ET^0_{wb} \end{cases} \tag{10}$$

According to this cooling model, entering air is in the mixed condition when the input value of the ET_{wb} lies close to ET^0_{wb}. In this case, Equation 10 should be used. However, when the input value of ET_{wb} is far from the critical ET^0_{wb}, Equation 9 is employed since the entering air can be in either in the wet or dry condition.

The Regression Method

In order to obtain the specific functional equation from the generic model, the key variable of ET^0_{wb} needs to be determined at a fixed (ET_{db}, CFM, OAT). It can be solved by applying the mathematical regression method to the real rating data in the wet coil condition and mixed condition. This study focuses only on linear regression, and one rooftop of manufacturer A (17.5 ton, four compressor stages) is taken as a typical example.

Visualization of Rating Data. Differing input values in the manual's rating data are given in Table 3.

Thus, for $\dot{Q}_T = f(ET_{wb}, CFM, OAT)$ and $SHR = f(ET_{wb}, ET_{db}, OAT, CFM)$, there are 36 (3*3*4) and 108(3*3*4*3) rating data points, respectively. In order to determine the order for the regression method or the specific mathematical equation that can give one output per input, a graphed curve or polyline showing the relationship of the output with a primary input is visualized while all other inputs are kept constant. A secondary variable input is chosen from the remaining inputs and graphed. Finally, one of the remaining inputs (excluding the primary and secondary inputs) is selected as a "horizontal input," or as the variable that differs when graphs are compared in a horizontal direction. If after excluding the above inputs some still remain, the vertical input is chosen as the differentiating variable and compared in a vertical direction. There are, at most, four inputs per function that express the changing relationship between the output and all individual inputs in a set of graphs. As shown in Figure 6, the resulting graphs are

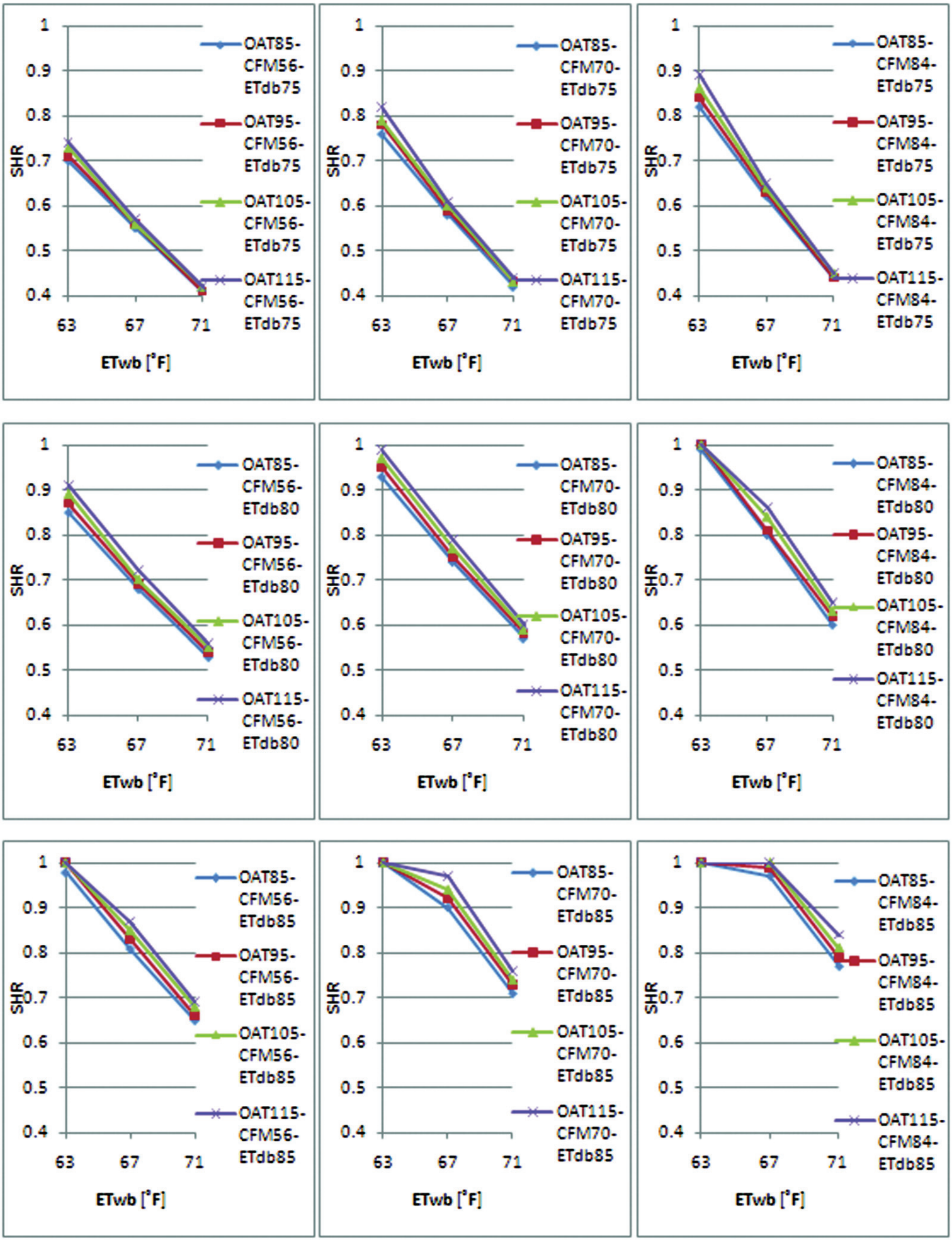

Figure 6a. Visualization of the relationship of output (*SHR*) with input (ET_{wb}).

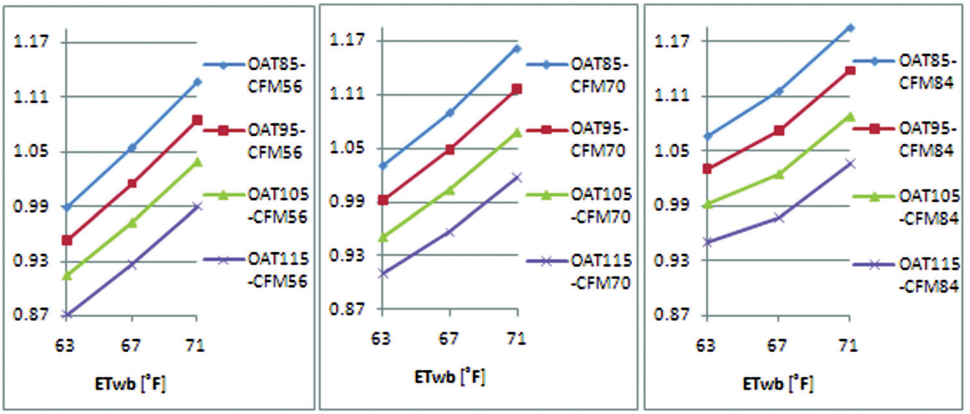

Figure 6b. Visualization of the relationship between the normalized output ($\dot{Q}_T$) and input (ET_{wb}).

arranged so that inputs that differ in value going horizontally and vertically are placed together and compared.

Using differing values for the secondary variable input and fixed values for the other inputs, a changing relationship between the output and primary input is revealed. From the relationship of $\dot{Q}_T$ with ET_{wb} given in Figure 6b, *OAT* can be chosen as a secondary input (i.e., each polyline shows the relationship of $\dot{Q}_T$ with ET_{wb}, and different lines in one graph have different *OAT* but the same *CFM* value). Graphs are aligned in a horizontal direction according to the change in *CFM*. Using the same methodology, $SHR = f(ET_{wb}, ET_{db}, CFM, OAT)$ can also be visualized, assuming ET_{wb}, *OAT*, *CFM*, ET_{db} is a primary variable, secondary variable, and variables of change in a horizontal or vertical direction, respectively. In Figure 6a, each polyline gives the relationship of *SHR* with ET_{wb}, so that differing lines in one graph have differing *OAT* values but the same (ET_{db}, *CFM*). As illustrated by the set in Figure 6a, graphs are aligned in a horizontal direction according to the change in *CFM* and in a vertical direction according to the change in ET_{db}. For each set of possible scenarios, different inputs can be chosen as the secondary variable, variable of change in a horizontal or vertical direction.

The following can be deduced from the graphs:

1. The relationship of *SHR* and ET_{wb} in the function $SHR = f(ET_{wb}, ET_{db}, OAT, CFM)$.
   a. When the ET_{db} is the fixed value, the *CFM* has a greater effect on *SHR* than *OAT*. The *SHR*s with a larger *CFM* value are greater than those with a smaller *CFM*. A *SHR* with a high *OAT* value increases when *CFM* and ET_{db} are kept constant.
   b. When the *OAT* is the fixed value, the ET_{db} has a greater effect on *SHR* than *CFM*. In most cases, *SHR*s that have a large ET_{db} are larger than those with a smaller ET_{db}. An *SHR* with a higher *CFM* is higher when *OAT* and ET_{db} are fixed values.
   c. When *CFM* is the fixed value, the ET_{db} has a greater effect on *SHR* than *OAT*. *SHR* values with a larger ET_{db} are greater than those with a smaller ET_{db}. An *SHR* with a higher *OAT* is also higher when *CFM* and ET_{db} are kept constant.
   d. The significance of the input's effect on the value of *SHR* is in the order of $ET_{db} > CFM > OAT$.

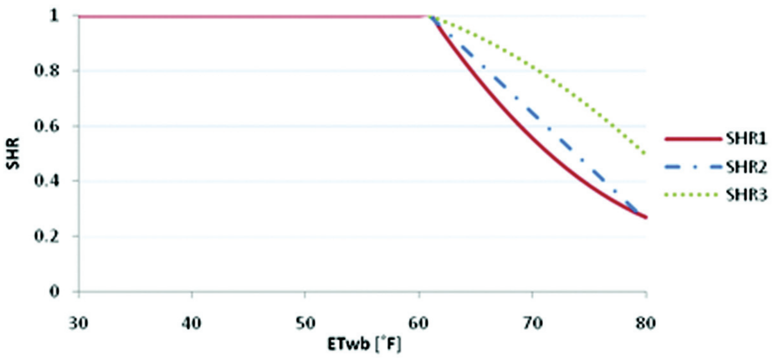

Figure 7a. Predicted relationship of output (*SHR*) with input (ET_{wb}).

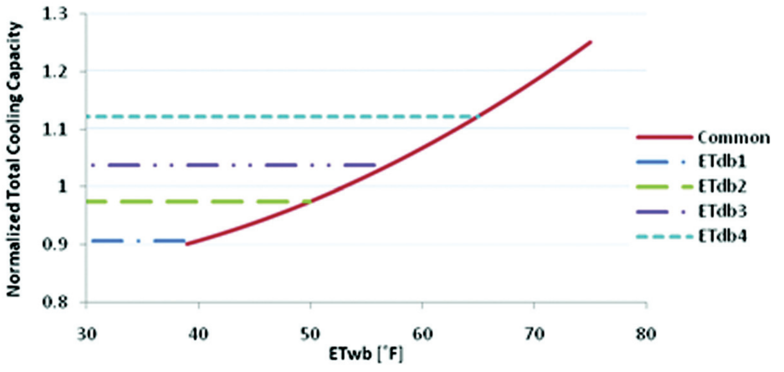

Figure 7b. Predicted relationship of the normalized output ($\dot{Q}_T$) with input (ET_{wb}).

e. The predicted relationship of *SHR* and ET_{wb} at a fixed (ET_{db}, *CFM*, *OAT*) is represented by the concave *SHR*1 curve (but not the *SHR*2 line or the convex *SHR*3 curve, as seen in Figure 7a).

2. The relationship between $\dot{Q}_T$ and ET_{wb} is the function $\dot{Q}_T = f(ET_{wb}, CFM, OAT)$.
   a. With a lower *CFM* value, ET_{wb} and $\dot{Q}_T$ lines are closer to forming a linear relationship (e.g., *CFM* = 5600 ft^3/min [2.64 m^3/s]). However, the linear relationship changes to a polyline with an increasing *CFM* value. In fact, a kinked point forms when *CFM* = 8400 ft^3/min (3.96 m^3/s).
   b. When the *CFM* is the fixed value, the slopes for the differing *OAT* conditions are similar, giving parallel *OAT* lines. However, the width of the space between these lines differs. It therefore follows that the higher the *OAT*, the wider the gap.
   c. When the *OAT* is the fixed value, the slopes of the larger *CFM* values are lower. With a higher *CFM*, the space between the lines is smaller. Thus, the $\dot{Q}_T$ value does not linearly change with *OAT* and *CFM*. The $\dot{Q}_T$ changes with *OAT* and/or *CFM*, and is higher when the *CFM* lowers and/or the *OAT* is higher.

   d. The predicted relationship between the $\dot{Q}_T$ and ET_{wb} at a fixed (CFM, OAT) is shown in Figure 7b. The solid curve is the common curve shared by all ET_{db} scenarios. This curve becomes a horizontal line at ET^0_{wb} when the ET_{wb} value decreases for each ET_{db} value at a fixed (CFM, OAT). Each ET_{db} value has a specific line. This line is high when the ET_{db} value is high and follows the order $ET_{db1} < ET_{db2} < ET_{db3} < ET_{db4}$.

3. Figure 5 shows the relationship of $\dot{Q}_T$ and SHR with ET_{wb} at a fixed (ET_{db}, CFM, OAT).

Determining the Regression Function Equation. The cooling mechanic ($\dot{Q}_T$, SHR) of any driving condition (ET_{db}, ET_{wb}, CFM, OAT) can be derived so long as the function equations of ($\dot{Q}_T$, SHR) with inputs (ET_{db}, ET_{wb}, CFM, OAT) for the wet-coil condition are known and the ET^0_{wb} determined.

1. Procedure for determining Equation 1 (i.e., $\dot{Q}_T = f(ET_{wb}, CFM, OAT)$) for wet-coil conditions using rating data.
   a. Use only wet-coil conditions (i.e., $SHR < 1$) as regression data.
   b. If the manufacturer's rating data provides a different $\dot{Q}_T = f(ET_{wb}, CFM, OAT)$ for each ET_{db}, take the average $\dot{Q}_T$ at these different ET_{db}. $\dot{Q}_T$ values at differing ET_{db} only differ by a small amount.
   c. Obtain the second-order polynomial regression equation (including the cross-terms) from the selected manufacturer's rating data.

2. Procedure for determining Equation 3 (i.e., $SHR = f[ET_{db}, ET_{wb}, CFM, OAT]$) in wet-coil conditions and mixed conditions using rating data.
   a. Use the manufacturer's ratings for the wet- and dry-coil conditions as regression base data to calculate the second-order polynomial regression equation (including the cross-terms) such that $SHR_0 = f_0(ET_{db}, ET_{wb}, CFM, OAT)$.

$$Calculated\ SHR_0 = \min(f_0[ET_{db}, ET_{wb}, CFM, OAT], 1)$$

   b. Calculate the relative error Rel_{SHR_0} = ($real\ SHR$ − $Calculated\ SHR_0$) / $real\ SHR$ for all data. If the absolute $ABS(Rel_{SHR_0}) < 0.04$, then SHR_0 is the required SHR value giving $SHR = SHR_0 = f_0(ET_{db}, ET_{wb}, CFM, OAT)$.
   c. If the absolute $ABS(Rel_{SHR_0})$ is greater than the value stated above, select all wet coil and the dry-coil data that have an $SHR_0 < 1$ (i.e., excluding dry-coil condition data with $SHR_0 \geq 1$) as the regression base data. Use this information to solve for the second-order polynomial regression equation (including the cross-terms), so that $SHR_1 = f_1(ET_{db}, CFM, OAT)$. Next, calculate the relative error, Rel_{SHR_1} = ($real\ SHR$ − $Calculated\ SHR_1$) / $real\ SHR$ for all data. If the absolute $ABS(Rel_{SHR_1}) < 0.04$ for all data, then SHR_1 is the required SHR value. If that value cannot be attained, repeat step 2 for $SHR_i (i > 1)$ until the $ABS(Rel_{SHR_i}) < 0.04$.
   d. If after several test runs a relative $ABS(Rel_{SHR_i}) < 0.04$ still cannot be attained, select the SHR_i test with the lowest maximum or average $ABS(Rel_{SHR_i})$. Fortunately, the correct SHR function is usually attained before the third trial run.

If data from the wet coil condition are only used as the regression base data, the calculated ET^0_{wb} from the regression equation should be separated from the actual ET^0_{wb}. This is because the actual ET^0_{wb} lies in the mixed condition, a situation unaccounted for in the regression base data. Dry coil condition data with $SHR_i < 1$ is considered as data for the mixed condition.

3. Find the ET^0_{wb} or transition point using data for the wet- and dry- coil conditions.
   a. At a fixed (ET_{db}, CFM, OAT), the equation $SHR = f(ET_{db}, ET_{wb}, CFM, OAT)$ is actually a quadratic equation of ET_{wb}, i.e.,

$$SHR = a \cdot ET^2_{wb} + b \cdot ET_{wb} + c \, ,$$

   where (a, b, c) are constants at a fixed (ET_{db}, CFM, OAT). Given $SHR = 1$, the ET^0_{wb} can be easily solved from the following equation:

$$a \cdot ET^2_{wb} + b \cdot ET_{wb} + c - 1 = 0 \, , \tag{11}$$

$$\text{i.e., } ET^0_{wb} = \frac{-b - \sqrt{b^2 - 4 \cdot a \cdot (c-1)}}{2 \cdot a}$$

Determining the Cooling Model

Thus, for any operating driving input (ET_{db}, ET_{wb}, CFM, OAT), the following cooling formula is obtained:

$$(ET_{db}, ET_{wb}, CFM, OAT)\begin{cases} ET_{wb} > ET^0_{wb}\begin{cases} \dot{Q}_T = f(ET_{wb}, CFM, OAT) \\ SHR = f(ET_{wb}, ET_{db}, OAT, CFM)(1 > SHR \ge 0) \\ \dot{Q}_S = SHR \cdot \dot{Q}_T \end{cases} \\ ET_{wb} \le ET^0_{wb}\begin{cases} \dot{Q}_T = \dot{Q}^0_T = f(ET^0_{wb} CFM, OAT) \\ SHR = 1 \\ \dot{Q}_S = \dot{Q}^0_T \end{cases} \end{cases} \tag{12}$$

where Equations 1 and 3 are obtained by a procedure (2.3.2) based on the manufacturer's rating data, and ET^0_{wb} is determined by Equation 11 at a fixed (ET_{db}, CFM, OAT).

Rooftop B17.5 (manufacturer B, 17.5 ton) was taken as a case study, and its detailed calculation procedure and original manual data are respectively attached in Appendix A and B.

MODELING VALIDATION AND COMPARISON

The performance of the cooling model was validated using both the manufacturers' rating data and experimental data. In terms of manufacturers' data, the main objective is to extrapolate and validate the model using the dry-coil rating data since these data are not used as the regression base data. There is a sufficient amount of manufacturer's rating and lab data to account for every operating condition of the actual packaged HVAC unit. The sensible cooling capacity includes both $\dot{Q}_T$ and SHR variables. In the model, the relative error of sensible cooling capacity is taken as an index of the overall error. The relative error of the sensible cooling capacity RErr_$\dot{Q}_S$ is defined as ([model $\dot{Q}_S$ − actual $\dot{Q}_S$] / actual $\dot{Q}_S$) and ([model $\dot{Q}_S$ − measured $\dot{Q}_S$] / measured $\dot{Q}_S$) for manufacturers' ratings and the experimental data, respectively.

Table 4. Statistics of Manufacturer A (7.5 ton)

Data Type	Sets of Data	Min	Max	Average	Absolute Average	Standard Deviation
Wet coil condition	98	−1.98%	1.67%	0.00007	0.0060	0.0075
Dry coil condition	10	0.33%	4.21%	0.0275	0.0275	0.0131
All	108	−1.98%	4.21%	−0.0026	0.0080	0.0114

Table 5. Statistics of Manufacturer B (7.5 ton)

Data Type	Sets of Data	Min	Max	Average	Absolute Average	Standard Deviation
Wet coil condition	117	−3.57%	2.94%	0.0002	0.0112	0.0136
Dry coil condition	75	−4.83%	−0.008%	−0.0256	0.0256	0.0094
All	192	−4.83%	2.94%	−0.0101	0.0168	0.0174

Selecting the Manufacturers' Data

The manufacturers' rating data were selected to best suit the conditions of the lab research. One rooftop of manufacturer A (7.5 ton) was chosen because they model those used in the experimental research facilities. One rooftop of manufacturer B (7.5 ton) was also selected for validation because its $\dot{Q}_T$ is a function of ET_{db}, i.e., $\dot{Q}_T = f(ET_{wb}, ET_{db}, OAT, CFM)$. Manufacturer B also provides ample ratings data for the dry-coil condition. The relative error of the sensible cooling capacity RErr_$\dot{Q}_S$ for the two manufacturers is shown in Tables 4 and 5.

As seen in the above two tables, the accuracy and precision of the cooling model for the wet-coil condition is high. Manufacturer A has ABS(RErr_$\dot{Q}_S$) < 2%, its average almost 0, and standard deviation 0.0075. Manufacturer B delivers an accuracy equivalent to manufacturer A, although its maximum ABS(RErr_$\dot{Q}_S$) is much higher (i.e., 3.57%). In dry-coil conditions, both the accuracy and precision of the cooling model for manufacturer B are also very high, although its maximum ABS(RErr_$\dot{Q}_S$) and averages are much higher than those in wet-coil conditions. Manufacturer A and B give the similar maximum ABS(RErr_$\dot{Q}_S$) of about 4.5%, with an average of 0.02 and standard deviation of 0.01. Therefore, the presented cooling methodology is adequate for manufacturers A and B, showing a strong robustness.

Experimental Measurement

Experimental data were acquired at a packaged HVAC (manufacturer A, 7.5 ton) installed in a psychrometric test room in Omaha, Nebraska. Figure 8 shows the lab's air system and the location of its sensors. T1, T2, and T3 are outside air, return air, and discharge air dry-bulb temperature sensors, respectively; H1 and H2 are outside-air and return-air relative humidity sensors, respectively; F is the supply airflow meter. We did not directly measure the thermal conditions of the mixed air (or entering air) because their measurements are inaccurate in many cases. We indirectly used the discharge air temperature (*DAT*) and the air temperature rise through the supply fan (ΔT) to obtain these thermal conditions for the entering air, following the logic given below.

Firstly, we have the following known variables: *OAT*, *OARH* (measured relative humidity of outside air), and ω_{OA} (humidity ratio of outside air, $\omega_{OA} = f\,[OAT, OARH]$); *RAT* (measured

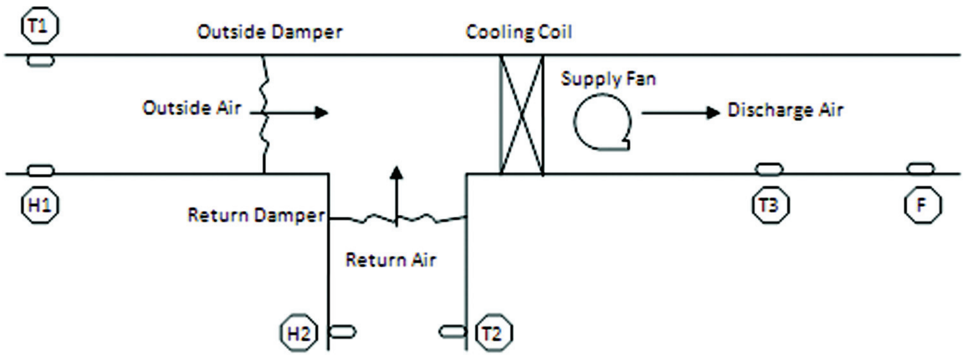

Figure 8. Air and sensor system of the lab.

dry-bulb temperature of return air), *RARH* (measured relative humidity of return air), and ω_{RA} (humidity ratio of return air, $\omega_{RA} = f\,[RAT, RARH]$); *DAT* (measured dry-bulb temperature of discharge air). From the above known variables, we need obtain the thermal conditions of entering air using Equations 13 and 14: ET_{db}, ω_{ET} (humidity ratio of mixed air), and then $ET_{wb} = f(ET_{db}, w_{ET})$.

$$ET_{db} = \alpha \cdot OAT + (1-\alpha) \cdot RAT \tag{13}$$

$$\omega_{ET} = \alpha \cdot \omega_{OA} + (1-\alpha) \cdot \omega_{RA} \tag{14}$$

However, there is still one unknown variable: α (outside air flow ratio), and we further suppose

$$\alpha = f(season, Damp\%, CFM)\ , \tag{15}$$

where *season* is the operation schedule (e.g., winter, summer) and *Damp%* is the damper position of the economizer. It doesn't matter whether the cooling and heating coils are closed or open. Thus, in order to obtain the thermal conditions of entering air, we only need know the value when both cooling and heating coils are closed.

When both cooling and heating coils are closed, $ET_{db} = DAT - \Delta T$. Combining the above equation with Equation 13, we obtain

$$\alpha = (DAT - RAT - \Delta T)/(OAT - RAT) \qquad \text{i.e., } \alpha = f(DAT, OAT, RAT, \Delta T)\ , \tag{16}$$

where

$$\Delta T = f(season, CFM)\ . \tag{17}$$

Further experiments were conducted to figure out the ΔT value at a fixed (*season*, *CFM*) as follows. At a fixed (*season*, *CFM*), we set a *Damp*% (e.g, *Damp*%1), and then changed different (*OAT*, *RAT*) to obtain different *DAT* values. That is, we had a set of data that satisfied Equation 16 at the same but unknown α and ΔT_1. ΔT_1 can then be solved by trial-and-error and best-fitting methods. In the same way, we chose a different *Damp*% (e.g., *Damp*%2) at the same fixed (*season*, *CFM*), and then obtained ΔT_2. After several *Damp*%s (e.g., *m*) were chosen, we obtained the average value (i.e., $\Delta T = [\Sigma_1^m \Delta T_i]/m$) and the standard deviation (i.e., $Std_{\Delta T} = \sqrt{\Sigma_1^m [\Delta T - \Delta T_i]^2/[m-1]}$) as the air temperature rise through the supply fan and ΔT's uncertainty measure at the fixed (*season*, *CFM*), respectively.

After ΔT was known at differing (*season*, *CFM*)s, Equation 16 was simplified to $\alpha = f(DAT, OAT, RAT)$, allowing us to obtain the α values (i.e., α_i, $1 \le i \le m$) using different lab data (*DAT*, *OAT*, *RAT*) at a fixed operating condition (*season*, *Damp*%, *CFM*). Then, we chose the average value ($\alpha = (\Sigma_1^m \alpha_i)/m$) as the value for the fixed operating condition (*season*, *Damp*%, *CFM*). The standard deviation ($Std_\alpha = \sqrt{\Sigma_1^m [\alpha - \alpha_i]^2/[m-1]}$) was taken as α's uncertainty measure. After the α value was known at differing operating conditions, we calculated the thermal conditions of the entering air using Equations 13 and 14.

The entering airflow rate was measured by the airflow meter installed at the end of the supply duct. The meter operates as follows. First, the velocity pressure is obtained by taking the measured total pressure minus the measured static pressure, then calculating the air velocity in the cross section of the supply duct. Finally, the airflow rate is obtained by multiplying the area of the cross section by the velocity.

Temperature sensors have a precision of ±0.5°F (0.3°C). Relative humidity sensors have a precision of ±1% when the dry-bulb temperature is above 68°F (20°C). In our tests, all measured dry-bulb temperatures (*OAT*, *RAT*) were above 68°F (20°C). The airflow meter has a precision of ±2%. We chose the summer season for cooling in our tests. ΔT was normally 1.7°F (0.9°C) and ΔT's uncertainty measure $Std_{\Delta T}$ was 0.12°F (0.07°C). Three damper positions (i.e., 0%, minimum, fully open) were used in our tests with their respective α values of around 20%, 53%, and 90% at differing *CFM*s. α's uncertainty measure Std_α was 3%.

All the sensors have a high precision for our tests. The calculated values directly impact the calculation of thermal conditions of the entering air, and is heavily influenced by the ΔT and *Damp*%. The ΔT values at different conditions are relatively close and *Damp*% also impacts the ΔT's calculation, so *Damp*% is one important factor in our thermal measurement of the entering air. The uncertainty of *Damp*% can come from hysteresis. A set damper position adjusted in the direction of increasing damper's opening is different from that adjusted in the direction of decreasing damper's opening due to hysteresis. Another important factor in the thermal measurement of the entering air is the packaged space (or small space) of the HVAC machine. Dampers are close to the area of mixed air, and the movement of air can affect *Damp*%. The packaged space of an economizer can lead to air leakage among outside air duct, return air duct, and supply air duct. For example, mixed air leaks to—and stays—in the outside air duct. The thermal measurement of the entering air will be impacted because the leakage changes the actual conditions of outside air, return air, and mixed air.

Experimental Data

Figure 9 gives the input range of the lab data and compares it to the data range supplied by the manufacturer. However, it excludes lab data for the higher wet-bulb temperature ET_{wb} and lower dry-bulb temperature ET_{db} for air entering the cooling coil. Notably, the experimental data range goes outside the range of the manufacturer's data, especially for the low *CFM*, low *OAT*, low ET_{wb} and high ET_{db} values. These values are used as a means to effectively extrapolate and validate the model.

Figure 10 shows the relationship between the measured sensible cooling capacity and the predicted sensible capacity. The experimental data lie at a close vicinity to each other within the

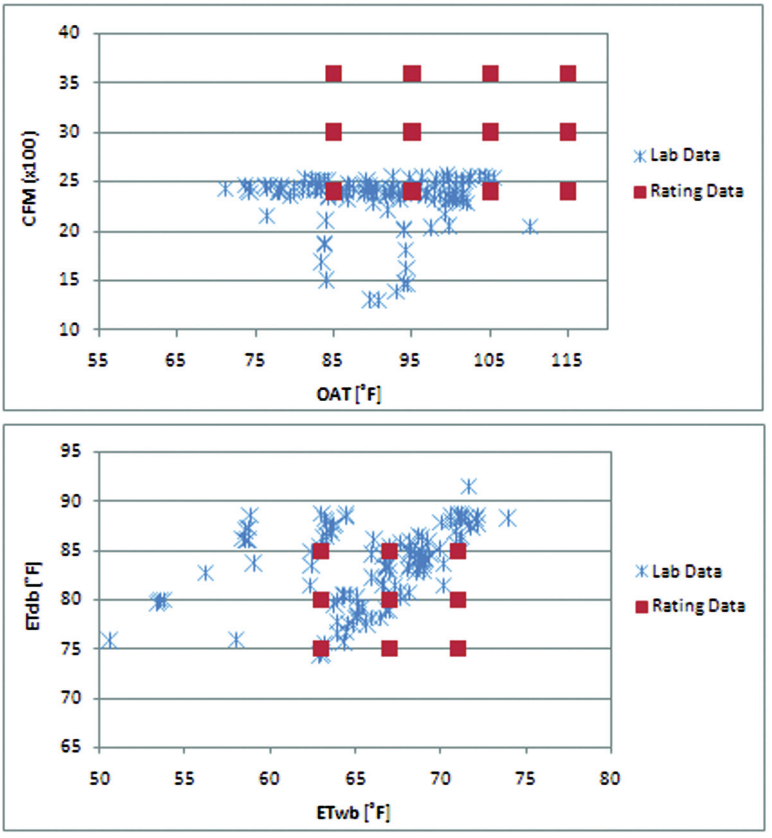

Figure 9. Experimental data range vs. the manufacturer's rating data range.

dashed lines, indicating a very low relative error. Table 6 shows that the relative error of the model's sensible cooling capacity and measured sensible cooling capacity is (–0.097, 0.0982). Table 6 also gives a statistical comparison of the RErr_$\dot{Q}_S$ for the experimental data. The relative error of RErr_$\dot{Q}_S$ has a maximum of 9.82%, a minimum of –9.7%, and the absolute average of below 0.0431. Hence, the cooling model performs well in both the wet-coil and dry-coil experimental data ranges.

CONCLUSION

Lab results are consistent with the manufacturers' rating data. Therefore, the cooling model presented in this paper has a high accuracy and robustness. The proposed generic modeling methodology can be easily implemented, requiring only information gathered from the manufacturers' rating data and simple calculations based on the mathematical regression method. It eliminates the need to select initial or rated conditions and perform complicated iteration calculations of the psychrometric properties, allowing nonspecialized users greater accessibility. Further, the new method will enable engineers to better facilitate the design, maintenance, and real-time automatic fault detection and diagnosis of packaged HVAC units. It is hoped that manufacturers can present data in as uniform a format as possible to strengthen effective communication and improve system implementation.

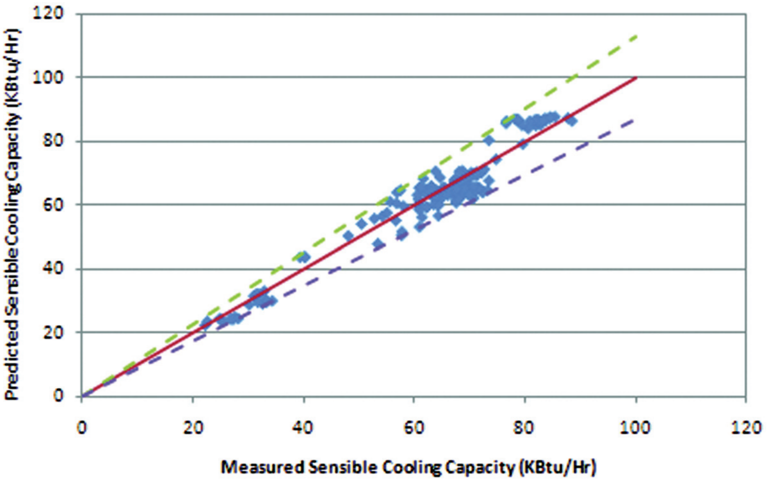

Figure 10. Measured vs. predicted sensible cooling capacity from the model.

Table 6. Statistics of Lab Data

Data Type	Sets of Data	Min	Max	Average	Absolute Average	Standard Deviation
Wet coil condition	95	–9.7%	9.82%	–0.0191	0.0426	0.0472
Dry coil condition	24	2.39%	9.56%	0.0398	0.0398	0.0148
All	119	–9.7%	9.82%	–0.0062	0.0431	0.0498

NOMENCLATURE

$\dot{Q}_T$ = total cooling capacity, kBtu/h (kW)
$\dot{Q}_S$ = sensible cooling capacity, kBtu/h (kW)
SHR = sensible heat ratio
CFM = flow rate of air entering evaporator, ft^3/min (m^3/s)
CFM_{Cond} = flow rate of air entering condenser, ft^3/min (m^3/s)
OAT = outside air temperature, °F (°C)
DAT = discharge air temperature, °F (°C)
RAT = return air temperature, °F (°C)
ΔT = air temperature rise through supply fan, °F (°C)
$OARH$ = outside air relative humidity, %
$RARH$ = return air relative humidity, %
ET = temperature of air entering evaporator, °F (°C)
T_{evap} = temperature of refrigerant entering evaporator, °F (°C)
n = number of cooling system's stage being on
ν = voltage, V
I = current, A
ADP = apparatus dew point, °F (°C)
C_{wb-h} = specific heat of wet coils, kBtu/(ft^3·°F) (kW/[m^3·°C])
C_{db-h} = specific heat of dry coils, kBtu/(ft^3·°F) (kW/[m^3·°C])
BF = bypass factor of evaporator
ω_{OA} = humidity ratio of outside air, lbm H_2O / lbm dry air (kg H_2O / kg dry air)
ω_{RA} = humidity ratio of return air, lbm H_2O / lbm dry air (kg H_2O / kg dry air)
ω_{ET} = humidity ratio of air entering evaporator, lbm H_2O / lbm dry air (kg H_2O / kg dry air)
α = outside airflow ratio
$Damp\%$ = damper position of economizer
$season$ = operation schedule (e.g., winter, summer)

Std_α	=	standard deviation of α
$Std_{\Delta T}$	=	standard deviation of ΔT
$\text{RErr}\_\dot{Q}_S$	=	relative error of sensible cooling capacity between model calculation and real value
Rel_{SHR_i}	=	relative error of SHR between ith iteration calculation and real value ($i \geq 0$)

Subscripts and Superscripts

$rated$	=	rated condition
wb	=	wet bulb
db	=	dry bulb
0	=	critical point
i	=	ith iteration calculation

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APPENDIX A
CALCULATION PROCEDURE AND ITS CALCULATION CASE OF ROOFTOP B17.5

1. Calculate SHR values for the 192 sets of data, i.e., $SHR = \dot{Q}_S/\dot{Q}_T$.
2. Determine $\dot{Q}_T = f(ET_{wb}, CFM, OAT)$ function for wet-coil condition from manual data.
   a. First, filter out the dry-coil condition data (i.e., SHR = 1) from the manufacturer's rating data, and then take the average $\dot{Q}_T$ for the remaining data with different ET_{db} values but with the same (ET_{wb}, CFM, OAT). In fact, $\dot{Q}_T$ at different ET_{db} only has a slight difference.
   b. Second, use the above data as regression data to obtain the two polynomial-order regression equation (including the cross-terms), i.e.,

$$\dot{Q}_T = -2.07941376\text{E}{+}02 + 2.96627293\text{E}+00 \cdot CFM - 5.09934779\text{E}{-}03 \cdot CFM^2$$
$$- 2.80837085\text{E}{+}00 \cdot OAT - 1.20456417\text{E}{-}02 \cdot OAT^2 + 1.13374693\text{E}{+}01 \cdot ET_{wb}$$
$$- 8.90324909\text{E}{-}02 \cdot ET_{wb}^2 + 3.31590430\text{E-}03 \cdot CFM \cdot OAT - 3.25207538\text{E}{-}02$$
$$\cdot CFM \cdot ET_{wb} + 5.95485478\text{E}{-}02 \cdot OAT \cdot ET_{wb}.$$

3. Determine $SHR = f(ET_{db}, ET_{wb}, CFM, OAT)$ function for wet-coil condition from manual data.
   a. First, using all of the manufacturer's rating data (including the dry- and wet-coil condition data) as regression base data to obtain the two polynomial-order regression equation

(including the cross-terms), i.e., $SHR_0 = f_0(ET_{db}, ET_{wb}, CFM, OAT)$, *Calculated* $SHR_0 = \min(f_0[ET_{db}, ET_{wb}, CFM, OAT], 1)$.

$$SHR_0 = 2.06138630\text{E+00} + 4.86175823\text{E–03} \cdot CFM - 2.31885687\text{E–05} \cdot CFM^2 - 6.03107125\text{E–03} \cdot OAT - 3.47628104\text{E–06} \cdot OAT^2 - 2.12623314\text{E–02} \cdot ET_{db} - 6.23810658\text{E–04} \cdot ET_{db}^2 + 1.42567593\text{E-03} \cdot ET_{wb} - 1.70365679\text{E–03} \cdot ET_{wb}^2 + 5.08243110\text{E–06} \cdot CFM \cdot OAT - 7.18371002\text{E–05} \cdot CFM \cdot ET_{db} + 8.47359069\text{E–05} \cdot CFM \cdot ET_{wb} + 2.65979398\text{E–05} \cdot OAT \cdot ET_{db} + 8.21949015\text{E–05} \cdot OAT \cdot ET_{wb} + 2.16712122\text{E–03} \cdot ET_{db} \cdot ET_{wb}$$

b. Second, calculate the relative error Rel_{SHR_0} = (*real SHR* – *Calculated* SHR_0) / *real SHR* for all data. If the absolute $ABS(Rel_{SHR_0}) < 0.04$ for all data, then SHR_0 is the needed *SHR*, i.e., $SHR = SHR_0 = f_0(ET_{db}, ET_{wb}, CFM, OAT)$. However, the case has the maximum Rel_{SHR_0} at 11%, so the following step (one more trial run) is needed.

c. Otherwise, select both the wet-coil data and the dry-coil data with their SHR_0 (i.e., filter out the dry-coil condition data with their $SHR_0 \geq 1$) as the regression base data to obtain the two polynomial-order regression equation (including the cross-terms), i.e., $SHR_1 = f_1(ET_{db}, ET_{wb}, CFM, OAT)$. Then calculate the relative error Rel_{SHR_1} = (*real SHR* – *Calculated* SHR_1) / *real SHR* for all data. If the absolute $ABS(Rel_{SHR_1}) < 0.04$ for all data, then SHR_1 is the needed *SHR*. If not, then repeat step 3 to obtain $SHR_i (i > 1)$ until $ABS(Rel_{SHR_i}) < 0.04$ where SHR_i is the needed *SHR*. The case has the following SHR_1 equation and its maximum Rel_{SHR_1} is 7.9%, so one more trial run is needed.

$$SHR_1 = 9.15278278\text{E–01} + 9.81014645\text{E–03} \cdot CFM - 1.65320650\text{E–05} \cdot CFM^2 - 3.17794263\text{E–03} \cdot OAT + 3.91291794\text{E–06} \cdot OAT^2 + 1.14217256\text{E–02} \cdot ET_{db} - 5.74152410\text{E–04} \cdot ET_{db}^2 - 1.58296172\text{E–02} \cdot ET_{wb} - 6.72975815\text{E–04} \cdot ET_{wb}^2 + 3.44307387\text{E–05} \cdot CFM \cdot OAT + 7.48326342\text{E–05} \cdot CFM \cdot ET_{db} - 2.08790216\text{E–04} \cdot CFM \cdot ET_{wb} + 1.86863800\text{E–04} \cdot OAT \cdot ET_{db} - 1.93730853\text{E–04} \cdot OAT \cdot ET_{wb} + 1.21717851\text{E–03} \cdot ET_{db} \cdot ET_{wb}$$

d. Filter out the dry-coil data with their $SHR_1 \geq 1$ and use the remaining data as the regression base data to obtain the two polynomial-order regression equation (including the cross-terms), i.e., $SHR_2 = f_2(ET_{db}, ET_{wb}, CFM, OAT)$ and then its corresponding Rel_{SHR_2}. The maximum Rel_{SHR_2} is 4%, so SHR_2 is the needed *SHR*.

$$SHR_2 = 8.13491256\text{E–01} + 1.12900295\text{E–02} \cdot CFM - 1.05332286\text{E–05} \cdot CFM^2 - 5.14903396\text{E–03} \cdot OAT + 1.62786936\text{E–05} \cdot OAT^2 + 5.17463820\text{E–02} \cdot ET_{db} - 3.97020976\text{E–04} \cdot ET_{db}^2 - 6.07627777\text{E–02} \cdot ET_{wb} + 4.18798335\text{E–04} \cdot ET_{wb}^2 + 3.91372605\text{E–05} \cdot CFM \cdot OAT + 1.20515835\text{E–04} \cdot CFM \cdot ET_{db} - 2.98347811\text{E–04} \cdot CFM \cdot ET_{wb} + 2.70113976\text{E–04} \cdot OAT \cdot ET_{db} - 2.97676815\text{E–04} \cdot OAT \cdot ET_{wb} + 8.76128835\text{E–05} \cdot ET_{db} \cdot ET_{wb}$$

e. If there is no $ABS(Rel_{SHR_i}) < 0.04$ after several runs, we can choose the SHR_i whose max value or average value of $ABS(Rel_{SHR_i})$ is minimal among these trial runs. Fortunately, we generally can obtain the *SHR* function before the third trial run for our case calculations, that is, SHR_2 or SHR_1 is the right solution.

4. Determine ET^0_{wb}.
   a. For a fixed (ET_{db}, CFM, OAT), the equation $SHR = f(ET_{db}, ET_{wb}, CFM, OAT)$ is actually a quadratic equation of ET_{wb}, i.e., $SHR = a \cdot ET^2_{wb} + b \cdot ET_{wb} + c$, where ($a$, b, c) are constants at a fixed (ET_{db}, CFM, OAT). Given $SHR = 1$, the ET^0_{wb} can be easily solved from the following equation:

$$a \cdot ET^2_{wb} + b \cdot ET_{wb} + c - 1 = 0$$

$$\text{i.e., } ET^0_{wb} = \frac{-b - \sqrt{b^2 - 4 \cdot a \cdot (c-1)}}{2 \cdot a}$$

All coefficients of the case follow:

$$a = 4.18798335\text{E}{-04}$$

$$b = -6.07627777\text{E}{-02} - 2.98347811\text{E}{-04} \cdot CFM - 2.97676815\text{E}{-04} \cdot OAT + 8.76128835\text{E}{-05} \cdot ET_{db}$$

$$\begin{aligned} c = {} & 8.13491256\text{E}{-01} + 1.12900295\text{E}{-02} \cdot CFM - 1.05332286\text{E}{-05} \cdot CFM^2 - 5.14903396\text{E}{-03} \\ & \cdot OAT + 1.62786936\text{E}{-05} \cdot OAT^2 + 5.17463820\text{E}{-02} \cdot ET_{db} - 3.97020976\text{E}{-04} \cdot ET^2_{db} \\ & + 3.91372605\text{E}{-05} \cdot CFM \cdot OAT + 1.20515835\text{E}{-04} \\ & \cdot CFM \cdot ET_{db} + 2.70113976\text{E}{-04} \cdot OAT \cdot ET_{db} \end{aligned}$$

5. Having obtained $\dot{Q}_T = f(ET_{wb}, CFM, OAT)$, $SHR = f(ET_{wb}, ET_{db}, OAT, CFM)$, and ET^0_{wb}, we can quickly determine the coil's condition and its cooling capacity for any operating driving inputs (ET_{db}, ET_{wb}, CFM, OAT) using Equation 12.

Appendix B: Rooftop B17.5 Original Data Table

		OAT [˚F]																							
		85						95						105						115					
		ET_{wb} [˚F]																							
CFM (X100)	ET_{db} [˚F]	61		67		73		61		67		73		61		67		73		61		67		73	
		MBH	SHC	MBH	SHC	MBH	SHC	MBH	SHC	MBH	SHC	MBH	SHC	MBH	SHC	MBH	SHC	MBH	SHC	MBH	SHC	MBH	SHC	MBH	SHC
63	75	195	171	213	127	224	83	182	164	206	124	219	80.1	169	158	195	119	213	76.1	155	151	181	113	204	71.8
	80	200	200	214	157	226	114	191	191	207	156	221	112	180	180	196	153	214	110	168	168	183	147	205	106
	85	211	211	217	188	229	140	204	204	209	188	223	139	195	195	199	186	216	139	184	184	187	182	206	137
	90	219	219	220	217	230	164	214	214	214	214	225	167	207	207	207	207	218	169	198	198	198	198	208	168
70	75	199	181	215	131	225	84.7	187	176	208	129	221	81.4	174	169	198	125	215	77.7	160	160	185	119	206	73.4
	80	206	206	217	164	228	116	198	198	210	164	222	114	188	188	200	162	216	113	176	176	187	157	207	110
	85	216	216	220	197	230	144	210	210	213	198	225	144	202	202	204	198	218	145	192	192	192	192	209	144
	90	224	224	224	224	233	172	219	219	219	219	227	173	212	212	212	212	221	177	204	204	204	204	211	177
77	75	203	191	217	135	226	86.1	192	187	211	138	222	82.9	178	178	201	131	216	79.2	165	165	188	126	208	75
	80	211	211	219	169	229	118	204	204	212	171	224	117	194	194	203	170	218	116	183	183	191	167	209	114
	85	220	220	222	203	232	147	214	214	216	207	226	148	207	207	207	207	220	150	197	197	197	197	211	150
	90	226	226	226	226	235	176	223	223	222	222	229	179	216	216	216	216	223	184	208	208	208	208	214	186
84	75	206	199	218	138	227	87.8	195	195	212	143	223	84.3	183	183	203	136	217	80.6	171	171	190	132	209	76.5
	80	214	214	221	174	230	120	208	208	214	178	226	120	199	199	205	178	219	119	188	188	194	176	211	117
	85	222	222	224	210	233	150	217	217	218	215	228	151	211	211	211	211	222	155	202	202	202	202	213	156
	90	229	229	229	229	236	181	225	225	225	225	231	184	220	220	219	219	225	190	212	212	212	212	216	193

NOTES:
1. All capacities shown are gross and have not considered indoor fan heat. To obtain net cooling subtract indoor fan heat.
2. MBH = Total Gross Capacity [kBtu/hr]
3. SHC = Sensible Heat Capacity [kBtu/hr]
4. ETdb, Entering Air Dry Bulb Temperature [˚F]
5. ETwb, Entering air Wet Bulb Temperature [˚F]