Robust AC Optimal Power Flow for Power Networks With Wind Power Generation

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***Abstract*—This letter presents a robust optimization-based AC optimal power flow (RACOPF) model for power networks with uncertain wind power generation. Numerical results on three test systems with uncertain wind power show that the RACOPF outperforms the robust DC-OPF model in the literature.**

***Index Terms*—Optimal power flow (OPF), robust optimization, second-order cone programming (SOCP), wind power.**

I. Introduction

DUE to the increasing uncertainty caused by the increasing penetration of the intermittent renewable energy sources (RESs), such as wind power, the traditional deterministic optimization-based optimal power flow (OPF) may not provide the optimal control strategy for the power system operation. A promising approach to model the OPF involving uncertainty is robust optimization (RO). An affinely adjustable robust OPF (AAROPF) model was proposed in [1] for power networks with RESs. It minimizes the total cost of the conventional generators to balance the load and RES output variations defined by the uncertainty sets. However, the DC power flow was considered in the AAROPF, which neglects transmission losses and may lead to an infeasible OPF solution.

This letter extends the AAROPF model to propose a RO-based AC-OPF (RACOPF) model for power networks with uncertain wind power. However, a robust AC OPF problem in general is intractable or noncomputable. To solve this problem, the AC power flow constraints in the RACOPF are relaxed using the second-order cone programming (SOCP) technique [2], [3] and the RACOPF is then converted into a mixed-integer SOCP (MISOCP) model using a robust counterpart approach [4], which can be solved efficiently by an existing solver, such as Gurobi [5]. Numerical results show that the converted RACOPF is more robust than the AAROPF.

II. RACOPF for Power Networks With Wind Power

Consider a typical power network, where $\mathcal{B}$ denotes the set of n buses, $\mathcal{L}$ denotes the set of l transmission lines, $\mathcal{G}(\mathcal{G} \subseteq \mathcal{B})$ denotes the set of g buses to which conventional generators are connected, and $\mathcal{W}(\mathcal{W} \subseteq \mathcal{B})$ denotes the set of w buses to which wind farms are connected. The transfer admittance between buses i and $k(i,k \in \mathcal{B})$ is $G_{ik} + jB_{ik}$. The complex voltage at bus $i(i \in B)$ is $V_i = e_i + jf_i$ and its amplitude is limited between $V_i^{\min}$ and $V_i^{\max}$. Let P_{Di} and Q_{Di} be the real and reactive powers of the load at bus $i(i \in \mathcal{B})$, respectively. Let P_{Gi} and Q_{Vi} be the real and reactive powers of the conventional generator at bus $i(i \in \mathcal{G})$ with the lower/upper limits $p_{Gi}^{\min}/p_{Gi}^{\max}$ and $q_{Vi}^{\min}/q_{Vi}^{\max}$, respectively. Notes that

$$P_{Gi} = 0, Q_{Vi} = 0, i \notin \mathcal{G}. \tag{1}$$

Let $\boldsymbol{P_W} = (P_{Wi}, i \in \mathcal{B})$ and $\boldsymbol{\Delta P_W} = (\Delta P_{wi}, i \in \mathcal{B})$ be the vector of the expected values of the predicted real power outputs of the wind farms and the vector of the differences between $\boldsymbol{P_W}$ and its bounds, respectively, where $P_{Wi} = 0$ and $\boldsymbol{\Delta} P_{Wi} = 0$ if $i \notin \mathcal{W}$. Then, a set $\mathcal{R}^W$ of the uncertain wind power output $\tilde{P}_{Wi}(\forall i \in \mathcal{B})$ is defined as:

$$\mathcal{R}^W(\Gamma^W, \boldsymbol{P_W}, \Delta\boldsymbol{P_W}) := \{\tilde{P}_{Wi} : \exists \zeta_i \in \mathbb{R}\ s.t. \|\zeta_i\|_\infty \le \Gamma^W$$
$$\tilde{P}_{Wi} \in [P_{Wi} - \Delta P_{Wi}\zeta_i, P_{Wi} + \Delta P_{Wi}\zeta_i], \forall i \in \mathcal{B}\} \tag{2}$$

where $\zeta_i (0 \le \zeta_i \le 1)$ is the degree of uncertainty of the power output of the wind farm at bus i and $\zeta_i = 0$ when $i \notin \mathcal{W}$; $\Gamma^W (0 \le \Gamma^W \le 1)$ is the uncertainty level of the total wind power generation of the system.

Let $C_i(P_{Gi}) = c_{2i}P_{Gi}^2 + c_{1i}P_{Gi} + c_{0i}$ be the quadratic cost function of the generator $i(i \in \mathcal{G})$. If the mean value of the uncertain prediction error of the real power generation of each wind farm is zero and $\wedge$ is the covariance matrix of the uncertain prediction errors of the real power generations of all w wind farms, then the expected generation cost is:

$$\sum_{i \in \mathcal{G}} C_i(P_{Gi}) + \sum_{i \in \mathcal{G}} c'_{2i}\beta_i^2 \tag{3}$$

where $c'_{2i} = \left[\sum_{j=1}^{w}\sum_{k=1}^{w} \wedge_{jk}\right] \times c_{2i}$ and β_i is the participation factor of the generator at bus $i(i \in \mathcal{G})$ [1].

The objective of the RACOPF is to minimize (3) subject to the constraints (1) and (4)–(15):

1) Power flow equation in the SOCP form

$$P_{Gi} - P_{Di} + \tilde{P}_{Wi} = G_{ii}c_{ii} + \sum_{j \in \mathcal{B}} [G_{ij}c_{ij} - B_{ij}s_{ij}], i \in \mathcal{B} \tag{4}$$

$$Q_{Vi} - Q_{Di} = -B_{ii}c_{ii} + \sum_{j \in \mathcal{B}} [-B_{ij}c_{ij} - G_{ij}s_{ij}], i \in \mathcal{B} \tag{5}$$

$$(V_i^{\min})^2 \le c_{ii} \le (V_i^{\max})^2, i \in \mathcal{B} \tag{6}$$

$$c_{ij} = c_{ji}, (i,j) \in \mathcal{L} \tag{7}$$

$$s_{ij} = -s_{ji}, (i,j) \in \mathcal{L} \tag{8}$$

$$c_{ij}^2 + s_{ij}^2 \le c_{ii}c_{jj}, (i,j) \in \mathcal{L}. \tag{9}$$

2) Bounds of active power of AC line flow

$$|G_{ij}(c_{ii} - c_{ij}) + B_{ij}s_{ij}| \le p_{Lij}^{\max}, (i,j) \in \mathcal{L}. \tag{10}$$

3) Bounds of generation and voltage

$$P_{Gi} + \beta_i \sum_{w \in \mathcal{W}} \Delta P_{Ww}\zeta_w \le p_{Gi}^{\max}, i \in \mathcal{G} \tag{11}$$

$$p_{Gi}^{\min} \le P_{Gi} + \beta_i \sum_{w \in \mathcal{W}} \Delta P_{Ww}\zeta_w, i \in \mathcal{G} \tag{12}$$

$$q_{Vi}^{\min} \le Q_{Vi} \le q_{Vi}^{\max}, i \in \mathcal{G} \tag{13}$$

$$(V_i^{\min})^2 \le c_{ii} \le (V_i^{\max})^2, i \in \mathcal{B}. \tag{14}$$

4) Given magnitude of voltage at the reference bus

$$c_{ii} = 1 \text{ when bus } i \text{ is the reference bus} \tag{15}$$

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where $c_{ii} = e_i^2 + f_i^2, c_{ij} = e_i e_j + f_i f_j, s_{ij} = e_i f_i - e_j f_i, i, j \in \mathcal{B}$; the value of $\beta_i (i \in \mathcal{G})$ can be fixed or variable. When β_i is a variable, an additional constraint $\sum_{i \in \mathcal{G}} \beta_i = 1$ is needed. A discussion of the participation factor can be found in [1], [6]

Define the vector of decision variables $\boldsymbol{x} = [P_{Gi}, Q_{Vi}, c_{ii}, c_{ij}, s_{ij}, \beta_k]\ (i, j \in \mathcal{B}, k \in \mathcal{G})$ and vector of uncertain parameters $\boldsymbol{w} = [\zeta_i](i \in \mathcal{B})$, the RACOPF model can be expressed as:

$$\min_{\boldsymbol{x}} \max_{\boldsymbol{w}} f(\boldsymbol{x}, \boldsymbol{w})$$
$$\text{s.t } g(\boldsymbol{x}, \boldsymbol{w}) \preccurlyeq 0 \quad \forall \boldsymbol{w} \in \mathcal{R}^W. \tag{16}$$

The RACOPF can be converted to a computable model using the explicit maximization method. For example, the constraint (11) involves uncertain parameters. According to the uncertainty set (2), (11) can be represented by a linear constraint (17) with a ∞-norm-bounded uncertainty:

$$P_{Gi} + \beta_i \sum_{w \in \mathcal{W}} \Delta P_{Ww} \zeta_w - p_{Gi}^{\max} \leq 0, \forall \zeta_w : |\zeta_w|_\infty \leq \Gamma^W \tag{17}$$

where $P_{Gi}, \beta_i \in \boldsymbol{x}$, $\zeta_w \in \boldsymbol{W}$, $i \in \mathcal{B}$ and $w \in \mathcal{W}$.

Since $\max_{|\zeta_w| \leq p} \beta_i \sum_{w \in \mathcal{W}} \Delta P_{Ww} \zeta_w = \|\beta_i \sum_{w \in \mathcal{W}} \Delta P_{Ww}\|_{p*}$, where $\|\cdot\|_{p*}$ denotes the dual p-norm and $1/p + 1/p^* = 1$ [4], (17) can be replaced with:

$$P_{Gi} + \|\beta_i \sum_{w \in \mathcal{W}} \Delta P_{Ww}\|_{p^*} - p_{Gi}^{\max} \leq 0. \tag{18}$$

In this letter, p is ∞. Thus, $\|\cdot\|_{p^*}$ is one norm. The constraint (18) is linear without uncertain parameters and is converted into the mixed-integer form using the big-M method [4].

All other constraints containing uncertain parameters in (16) can be replaced with linear constraints without uncertainty and converted into the mixed-integer form in a similar way. The RACOPF (16) is therefore reformulated in the following form:

$$\min_{\boldsymbol{x}} \hat{f}(\boldsymbol{x})$$
$$\text{s.t } \hat{g}(\boldsymbol{x}) \preccurlyeq 0. \tag{19}$$

The new model (19) does not contain any uncertainty and is a MISOCP problem, which can be solved efficiently.

III. Numerical Results

The IEEE 14-bus, IEEE 118-bus, and 2736-bus systems are used for simulation. All data of the test systems can be found in [1]. The wind power penetrations relative to the total load for the three systems are 30.89%, 23.57%, and 4.43%, respectively. The reformulated RACOPF (19) is programmed in MATLAB2013a and solved using Gurobi [5] on a 3.4-GHz computer with a 16-G RAM. The maximum CPU times of solving (19) for the three systems with different uncertainty levels are 0.04 s, 0.3 s, and 100 s, respectively.

Table I compares the percentage increases in the optimal objective function values of the AAROPF and the RACOPF over the nominal deterministic case ($\Gamma^W = 0$) versus Γ^W when $\beta_i (i \in \mathcal{G})$ is fixed or variable. The optimal solutions of the RACOPF are more robust than those of the AAROPF, because for the same uncertainty level and β_i, the percentage increase of the optimal objective function value of the RACOPF is always lower than that of the AAROPF. In both models, a lower optimal objective function value is gained when β_i is variable as expected. Moreover, a larger power network has lower percentage increases in the optimal objective function value due to the increased flexibility in the redispatch through s β_is$(i \in \mathcal{G})$ and the lower wind power penetration level.

Next, the expected generation costs of the RACOPF and the AAROPF are compared using Monte Carlo simulations, where all β_is$(i \in \mathcal{G})$ are variable, and 50000 scenarios of the uncertain wind power generations are generated for each test system by using a multivariate normal distribution with zero means and a covariance matrix $\wedge$ given in [1]. In each scenario, the wind power is known; the generations of the RACOPF and the AAROPF are adjusted for real-time dispatch by solving a traditional AC-OPF problem using a primal-dual interior point method, where β_is$(i \in \mathcal{G})$ are obtained from the optimal solutions of the RACOPF and AAROPF, respectively. Rows 2–4 of Table II presents the percentage decreases in the expected generation cost of the RACOPF with respect to the AAROPF over the 50000 scenarios for different wind power uncertainty levels, where the realizations of the uncertain wind power generations in the 50000 scenarios have been covered by the RO models. The result in Row 5 is similar to that in Row 3, but the maximum variation of wind power generations in the 50000 scenarios is 1.5 times the uncertainty level considered by the RO models. In other words, not all of the realizations of the uncertain wind power generations in the 50000 scenarios are covered by the RO models. As a result, wind spillage or load curtailment may occur. The last row presents the percentage decrease of the wind spillage or load curtailment scenarios using the RACOPF with respect to the AAROPF over the 50000 scenarios. The RACOPF always gains a lower expected generation cost than the AAROPF. The improvement is more significant for a larger power network. The results show that the RACOPF outperforms the AAROPF.

TABLE I
Comparison of Percentage Increase in Objective Function Value of AAROPF and RACOPF for Different Levels of Uncertainty

	IEEE 14-bus				IEEE 118-bus				2736-bus			
	fixed β		variable β		fixed β		variable β		fixed β		variable β	
Γ^W	*	#	*	#	*	#	*	#	*	#	*	#
0	0	0	0	0	0	0	0	0	0	0	0	0
0.1	1.42	0.04	1.40	0	0.40	0.05	0.43	0.04	0.18	0.15	0.16	0.11
0.2	2.85	0.26	2.80	0	0.87	0.09	0.82	0.05	0.35	0.29	0.32	0.14
0.3	4.28	0.40	4.21	0	1.44	0.14	1.18	0.06	0.53	0.43	0.48	0.32
0.4	5.72	0.59	5.62	0	2.12	0.20	1.57	0.06	0.71	0.58	0.64	0.46
0.5	7.17	2.15	7.05	0.98	2.92	0.25	1.97	0.06	0.89	0.72	0.80	0.63
0.6	8.61	3.70	8.47	2.32	3.83	0.57	2.36	0.22	1.06	0.86	0.96	0.79
0.7	10.07	5.26	9.91	3.68	4.85	1.21	2.76	0.79	1.24	0.99	1.13	0.92
0.8	11.53	6.82	11.35	5.04	5.98	1.86	3.15	1.37	1.42	1.14	1.29	1.04
0.9	12.99	8.38	12.79	6.40	7.23	2.50	3.55	1.95	1.59	1.28	1.45	1.17
1	--	9.94	14.25	7.77	8.59	3.15	3.94	2.52	1.77	1.42	1.61	1.28

*: AAROPF; #: RACOPF; --: No solution.

TABLE II
Percentage Decrease in Expected Objective Function Value of RACOPF With Respect to AAROPF for Different Uncertainty Levels

Γ^W	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
14-bus	0.46	0.45	0.44	0.44	0.43	0.41	0.41	0.41	0.41	0.41	0.41
118-bus	1.54	0.54	0.71	0.59	0.58	0.57	0.56	0.56	0.56	0.47	0.48
2736-bus	1.83	1.03	1.04	1.05	1.07	1.10	1.05	1.12	1.11	1.07	1.15
118-bus	0.63	0.34	0.39	0.39	0.38	0.4	0.36	0.36	0.36	0.35	0.38
(1.5 Γ^W)	0.3	0.1	0.2	0.2	0.2	0.1	0.2	0.1	0.1	0.1	0.1

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