

# Incipient Bearing Fault Detection via Wind Generator Stator Current and Wavelet Filter

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**Abstract**—Bearing faults constitute a significant portion of all faults in rotating machines, including wind turbine generators (WTGs). Current-based bearing fault detection has significant advantages over traditional vibration-based methods in terms of cost, implementation, and system reliability. This paper proposes a new wavelet filter-based method for incipient bearing fault detection using electric machine stator currents. The proposed method can dramatically increase the signal-to-noise ratio (SNR) of the bearing fault related signals in the stator current samples. The normalized energy of the wavelet-filtered stator current signals is mainly related to bearing faults and is applied as the index for bearing fault detection. Experiments are carried out for an induction machine with developed bearing faults; the results show that the proposed method is effective to detect the bearing faults at an early stage.

## I. INTRODUCTION

Bearing faults constitute a significant portion of all faults in rotating machines, e.g., wind turbine generators (WTGs). For example, 40% of the faults of induction machines, which are commonly used for wind energy conversion, are bearing faults [1]-[2]. These bearing faults usually do not occur instantaneously but are developed gradually over time. It is highly desired to detect the bearing faults at an early stage and repair or replace the faulted bearing(s) to prevent catastrophic damages of the WTG systems.

Conventional bearing fault detection techniques require additional expensive mechanical sensors and data acquisition equipment to implement [3]. The most commonly used sensors are vibration sensors, such as accelerometers. These sensors are mounted on the surface of WTG components, which are situated on high towers and are difficult to access during WTG operation. Moreover, the sensors and equipment are inevitably subject to failure, which could cause additional problems with system reliability and additional operating and maintenance costs. According to statistical data reported in [4], sensor failures constitute more than 14% of failures in WTG systems, while more than 40% of failures are linked to the failure of sensors. Therefore, it is desirable to develop a nonintrusive, low-cost, more effective, and more reliable technology for online bearing fault detection. Current-based (mechanical-sensorless) fault detection techniques, in fact, meet these requirements, because they usually do not require additional sensors above those already used for monitoring,

controls, and protection of the WTG systems. Moreover, current signals are reliable and easily accessible from the ground. Therefore, current-based fault detection techniques have great economic benefits and potential to be adopted by industry.

According to different stages of the fault development process, bearing faults can be categorized as follows: 1) single-point defects, which typically occur at the very late stage of the bearing life or due to severe system failures, and 2) generalized roughness, which occurs when the bearing starts to degrade while it is still operable [5]. Much research effort has gone into the detection of single-point defects, where the characteristic fault frequencies are clear indicators for a present damage [6]. In fact, generalized roughness faults have also been observed in a significant number of cases of failed bearings from various industrial applications [5]. This type of faults exhibit degraded bearing surfaces, but not necessarily distinguished defects. However, little research has been done on detection of incipient generalized roughness faults, which will be the objective of this paper.

The major challenge in current-based bearing fault detection is that the fault related signals in the current measurements are weak and contaminated by dominant components, such as the fundamental component of the current measurements. This challenge becomes more serious when detecting generalized roughness bearing faults for WTG systems because these faults usually do not have a characteristic frequency. Wavelet transform [7] offers a powerful tool for feature extraction, data compression, and noise reduction in signal processing for nonstationary signals. Wavelet transform has been applied to detect characteristic frequencies of the faults of electric machines [8]-[10]. For the detection of generalized roughness bearing faults, which have no characteristic frequencies, a wavelet filter can be designed to filter out the dominant components in the current measurements that are irrelevant to bearing faults to discover the subtle fault signature. This however has not been reported by anybody yet.

This paper proposes a novel wavelet filter-based method for incipient bearing fault detection using inductor machine stator currents. The proposed wavelet filter is based on the discrete wavelet transform (DWT) and wavelet shrinkage [11]. The latter is a classical algorithm for noise elimination.

The remaining sections of the paper are organized as follows. Section II describes the proposed method. Section III presents the experimental setup and results to validate the proposed method for incipient bearing fault detection of an induction machine. The proposed method is compared with a Wiener filter-based stator current noise cancellation algorithm [12] developed by one of the authors of this paper at two different load conditions. Results show that the proposed wavelet filter-based method is effective for incipient bearing fault detection using machine current measurements.

## II. STATOR CURRENT WAVELET FILTER

### A. Components of Stator Currents

In frequency domain, the dominant components of the stator currents of an induction machine are the fundamental-frequency component and its multiple harmonics, e.g., the eccentricity, slot, and saturation harmonics, and other components from unknown sources including environmental noise [13]. These dominant components are not related to the bearing faults. In this sense, they are treated as *noise* in the bearing fault detection problem. To discover the bearing fault signature, i.e., the *fault signal*, in the stator current, it is desired to remove those dominant noise components from the stator current.

Since the generalized roughness faults do not have characteristic frequencies, traditional frequency-domain analysis-based methods are not effective to detect these faults. In this paper, the energy of the bearing fault signal in the stator current is extracted by using a wavelet filter, where the energy is defined as the square of the signal. Because the vibration of an electric machine is positively correlated to the degradation of the bearing, the magnitude of the energy of the bearing fault signal indicates the physical condition of the bearing. If the amplitude of the energy remains at a high level or vibrates with a large magnitude, it means the degradation of bearing and maintenance is required.

### B. Wavelet Decomposition

The continuous wavelet transform (CWT) [14] of a time-domain signal  $f(t)$  is given by:

$$CWT(a,b) = \left| a^{-1/2} \right| \int f(t) \cdot \psi\left(\frac{t-b}{a}\right) \cdot dt \quad (1)$$

where  $\psi$  is a wavelet function;  $a$  is a scaling parameter; and  $b$  is a time shifting parameter. For incipient bearing fault detection of WTG systems, the DWT [14] is applied by discretizing (1) and the result is given by:

$$DWT(m,n) = \left| a^{-m/2} \right| \int f(t) \cdot \psi(a_0^{-m}t - nb_0) \cdot dt \quad (2)$$

where  $m$  and  $n$  are integers;  $a_0 > 1$  and  $b_0 > 0$  are constant.

In the DWT, a wavelet function is associated with a scaling function. The wavelet function and the scaling function are finite vectors. The original data is decomposed into trend sub-signals by the scaling function and into fluctuations by the wavelet function. The wavelet decomposition is recursive, as shown in Fig. 1. This is known as multiresolution analysis

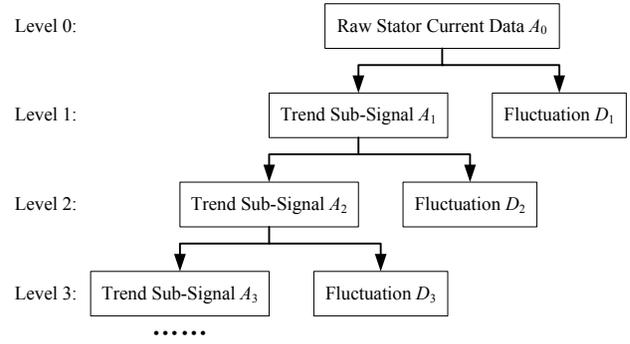


Fig. 1. The schematic diagram of wavelet decomposition.

[15]. Each of the trend sub-signals and fluctuations contains the time-domain features of the original data in a finite frequency band.

Assume the wavelet function is  $W(x) = [w_1, w_2, \dots, w_{2k}]$  and the scaling function is  $V(x) = [v_1, v_2, \dots, v_{2k}]$ , where  $k$  is a positive integer. The wavelets and scaling signals need to be generated firstly for the wavelet decomposition. The wavelets  $W_{i,m}$  are:

$$W_{i,m} = [0, \dots, 0, w_1, w_2, \dots, w_{2k}, 0, \dots, 0], m = 1, \dots, N_{i-1}/2 - k + 1 \quad (3)$$

$$W_{i,m} = [w_{2j+1}, \dots, w_{2k}, 0, \dots, 0, w_1, \dots, w_{2j}], m = N_{i-1}/2 - k + 2, \dots, N_{i-1}/2 \quad (4)$$

where  $i = 1, 2, 3, \dots$  is the level of the wavelet decomposition in Fig. 1;  $j$  is a positive integer, which is smaller than  $k$ ; the length of  $W_{i,m}$  is  $N_{i-1}$ ;  $w_1$  is the  $(2m-1)$ th element of  $W_{i,m}$  in (3) and (4). The scaling signals  $V_{i,m}$  are:

$$V_{i,m} = [0, \dots, 0, v_1, v_2, \dots, v_{2k}, 0, \dots, 0], m = 1, \dots, N_{i-1}/2 - k + 1 \quad (5)$$

$$V_{i,m} = [v_{2j+1}, \dots, v_{2k}, 0, \dots, 0, v_1, \dots, v_{2j}], m = N_{i-1}/2 - k + 2, \dots, N_{i-1}/2 \quad (6)$$

where the length of  $V_{i,m}$  is  $N_{i-1}$ ;  $v_1$  is the  $(2m-1)$ th element of  $V_{i,m}$  in (5) and (6).

At each level of the wavelet decomposition, the value  $d_{i,m}$  of each element of the fluctuation  $D_i = (d_{i,1}, d_{i,2}, \dots, d_{i,m})$  is [7]:

$$d_{i,m} = A_{i-1} \cdot W_{i,m}, m = 1, \dots, N_{i-1}/2 \quad (7)$$

where  $i = 1, 2, 3, \dots$  is the level of the wavelet decomposition in Fig. 1;  $A_{i-1}$  are the decomposed signals of length  $N_{i-1}$  at level  $i-1$ ;  $W_{i,m}$  are the wavelets at level  $i$  generated from the wavelet function by using (3) and (4). The value  $a_{i,m}$  of each element of the trend sub-signal  $A_i = (a_{i,1}, a_{i,2}, \dots, a_{i,m})$  is [7]:

$$a_{i,m} = A_{i-1} \cdot V_{i,m}, m = 1, \dots, N_{i-1}/2 \quad (8)$$

where  $V_{i,m}$  are the scaling signals at level  $i$  generated from the scaling function by using (5) and (6).

The performance of the DWT depends on the wavelet function chosen for decomposition. In this research, the Coiflet wavelet is applied due to its feature of vanishing moments. The vanishing moments of a wavelet function means that several moments of the wavelet function are zero. The vanishing moments of the Coiflet wavelet is designed not only in the wavelet function but also in the scaling function. The following equations illustrate such a feature for a continuous Coiflet wavelet [14]:

$$\int V(x) \cdot dx = 1; \quad (9)$$

$$\int x^l \cdot W(x) \cdot dx = 0, \text{ for } l = 0, 1, \dots, L-1; \quad (10)$$

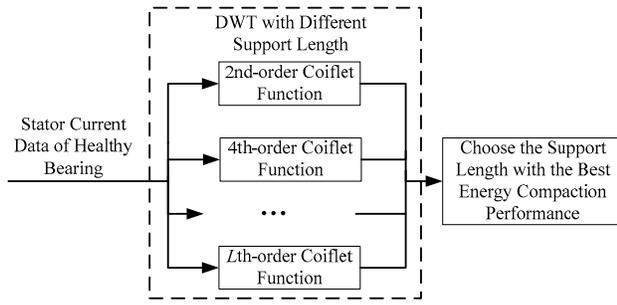


Fig. 2. The schematic diagram of pretreatment for the wavelet filter.

$$\int x^l V(x) dx = 0, \text{ for } l = 1, 2, \dots, L-1; \quad (11)$$

$$N = 3L-1 \quad (12)$$

where  $W(x)$  is the wavelet function;  $V(x)$  is the scaling function;  $L$  is the order of the Coiflet wavelet;  $N$  is the support length of the Coiflet wavelet. The support length measures the effective width of a wavelet function. Equations (10) and (11) give the vanishing moments in both the wavelet function and the scaling function of Coiflet, respectively. Because of this feature, the Coiflet wavelet is of symmetry and compactness for numerical analysis applications [16]. In this research, the dominant noise components in the stator current that are irrelevant to the bearing fault needs to be maximally compacted. Therefore, the Coiflet wavelet is a good candidate to implement such compaction.

### C. Choosing the Support Length for Wavelet Functions

The support length is an important parameter of a wavelet function. It determines the capability of compacting energy of a wavelet function in the DWT. The Coiflet wavelet functions with different support lengths are used to design the wavelet transform to maintain a close match between the trend values and the original signal values [7]. However, there are no rules for selecting the support length of the Coiflet wavelet in the DWT. Therefore, a pretreatment method is proposed to choose the support length of the wavelet function in this paper.

When the bearing is in healthy condition, the Coiflet wavelets of different support lengths are applied to decompose the stator current data. The one that can compact the largest energy of the stator current to a certain percent (e.g., 5%) of the whole length of the data is selected as the fixed support length of the Coiflet wavelet function for fault detection. The resulting Coiflet wavelet is assumed to have the most powerful capability to compact the dominant noise components of the stator current to a sub-signals through the DWT. The schematic diagram of the pretreatment is illustrated in Fig. 2, where  $L$  is an even integer.

### D. The Proposed Wavelet Filter

The proposed wavelet filter is based on the DWT and wavelet shrinkage. The DWT is used to decompose the stator current into different components; and the wavelet shrinkage works in a similar way to an adaptive notch filter to remove

the dominant noise components from the stator current. The resulting filtered signal is mainly related to the bearing fault.

The wavelet shrinkage is a traditional method for filter design [11]. In this paper, since the bearing fault signature in raw stator current samples is subtle and broad-band, the wavelet shrinkage should be operated to cancel the dominant noise components, which are irrelevant to the bearing faults. The proposed wavelet filter is implemented as follows:

1) Decompose a batch of stator current samples  $F(n) = [f(1), f(2), \dots, f(N)]$  by using the DWT with a Coiflet wavelet and the result is  $C_f(n) = [c(1), c(2), \dots, c(N)]$ , where  $n = 1, 2, \dots$ .

2) Calculate the energy of  $C_f(n)$  and the result is  $E(n) = [e(1), e(2), \dots, e(N)]$ , where  $e(i) = c^2(i)$  for  $i = 1, 2, \dots, N$ . Sort the elements of the energy vector  $E(n)$  from the minimum to the maximum value and the result is  $E_p(n)$ .

3) Choose a constant number  $h$ , which is approximately 10% of  $N$ ; calculate the sum from  $E_p(1)$  to  $E_p(h)$  and the result is  $E_d$ , which is assumed to be the total energy of the weak-energy components in the stator current samples  $F(n)$ .

4) Calculate the sum from  $E_p(1)$  to  $E_p(N)$  and the result is  $E_s$ , which is the total energy of the stator current samples  $F(n)$ .

5) Normalizing the energy  $E_d$  in terms of the total energy  $E_s$ , yields  $I = E_d/E_s$ , where  $I$  is defined as an index of the energy of the components, i.e., the fault signal, related to the bearing fault in the stator current samples.

The amplitude of  $I$  indicates the physical condition of the bearing. If the amplitude of  $I$  remains at a high level or vibrates with a large magnitude, it means the degradation of the bearing and maintenance is required.

The DWT decomposes the original signal into two parts: trend sub-signal and fluctuation. The high energy components of the original signal are compacted to its trend sub-signal; while the fluctuation only contains the weak energy components. This is called the compaction of energy, which is one of the main characteristics of the DWT [7]. The proposed wavelet filter can eliminate the high energy components in the stator current, which are the dominant noise components irrelative to the bearing faults. Therefore, the fault signature can be discovered from the wavelet filtered stator current. As the physical condition of the bearing becomes worse and worse, the amplitude of the energy of the

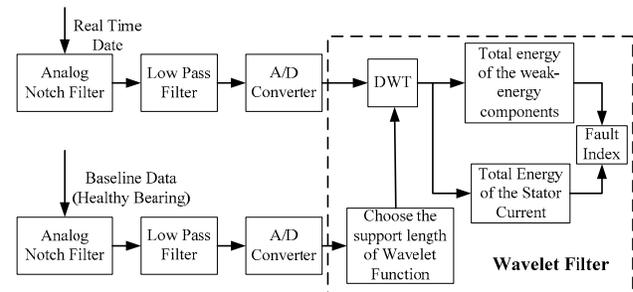


Fig. 3. The schematic diagram of the proposed wavelet filter-based bearing fault detection algorithm.

fault signal becomes more and more significant, which results in the increase of  $I$ . There are two reasons to normalize the energy  $E_d$  in terms of the total energy  $E_s$  to obtain  $I$ . First, the normalization can eliminate the interferences from the data acquisition equipment and electric machine itself. For instance, the ratio of an A/D converter may drift with temperature; the electric machine working at different cooling conditions may introduce variable stator current patterns. Furthermore, WTGs are usually operated under variable operating conditions due to the variations of wind sources. Only the normalized energy is reasonable to be used as a fault index in variable operating conditions.

The schematic diagram of the proposed wavelet filter-based bearing fault detection algorithm is illustrated in Fig. 3. The notch filters are used to remove the fundamental-frequency component of the stator current. The low-pass filters are used for anti-aliasing. The baseline data are the first several samples obtained from the healthy bearing, as it is assumed that the bearing is healthy initially. These baseline data are used to determine the support length of the Coiflet wavelet.

#### E. Validating the Wavelet Filter Using Artificial Data

The proposed method is firstly validated by using artificial data. The artificial data consist of two parts. One part emulates the narrow-band dominant noise components in the stator current that are irrelevant to bearing faults, defined as:

$$g(n) = \sum_{m=1}^M A_m \sin(\omega_m n + \theta_m) \quad (13)$$

where  $g(n)$  is the fault-irrelevant noise components;  $A_m$ ,  $\omega_m$ , and  $\theta_m$  are the amplitude, angular frequency, and phase angle of each component. In this paper, four ( $M=4$ ) different fault-irrelevant noise components with the frequencies of 60 Hz, 120 Hz, 180 Hz, and 240 Hz are used. They emulate the fundamental stator current and its multiple harmonics. The broad-band bearing fault component is assumed to be Gaussian noise. A Gaussian noise with a higher magnitude means the worse physical condition of the bearing. Therefore, the whole emulated stator current  $s(n)$  is:

$$s(n) = g(n) + \text{Gaussian}(n) \quad (14)$$

where  $\text{Gaussian}(n)$  is the Gaussian noise.

One hundred replications of  $s(n)$  are generated to emulate the degradation of the bearing condition through adding a Gaussian noise in each replication. The signal-to-noise ratio (SNR) of  $s(n)$  reduces linearly from 39.9930 dB for the first replication to 39.2758 dB for the last replication. The reduction of the SNR leads to the increase of the Gaussian noise in  $s(n)$ . In order to simulate the variable-speed operation of a WTG, the amplitude  $A_m$ , angular frequency  $\omega_m$ , and phase angle  $\theta_m$  of  $g(n)$  in (13) are randomly varied in a range of  $\pm 0.2$ ,  $\pm 20\%$ , and  $\pm \pi$  of their base values, respectively.

When applying the proposed wavelet filter to the artificial data, the 6th order wavelet functions, Coiflet3, is used, and the parameter  $h$  in Step 3 of the proposed wavelet filter is 5000. The simulation results are shown in Fig. 4. Fig. 4(a)

shows the power spectrum densities (PSDs) of four replications, which obviously have different dominant frequencies with each other. Fig. 4(b) shows the wavelet filtered results of the normalized energy of the emulated fault component, i.e., the Gaussian noise, which clearly shows that the energy of the broad-band fault component increases with the number of replications. These results demonstrate the effectiveness of the proposed method, namely, the wavelet filter is able to detect the increasing energy of the broad-band fault component.

### III. EXPERIMENTAL RESULTS

Experiments are carried out to validate the proposed method for incipient bearing fault detection of an induction machine. The configuration of the experimental setup is shown in Fig. 5. The induction machine is driven by a DC motor, which emulates the dynamics of a wind turbine. The

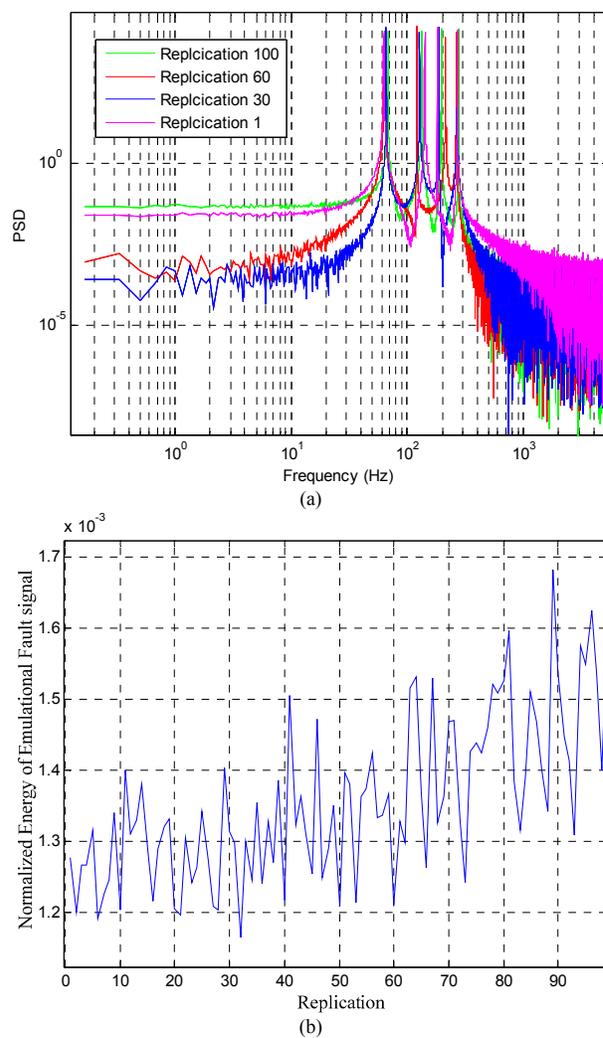


Fig. 4. Validation of the wavelet filter using artificial data: (a) PSDs of four replications, (b) energy of the fault signal vs. number of replications.

stator current of one phase is sampled for bearing fault detection. This setup uses a shaft current [17], which flows through the test bearing to accelerate the aging process of the bearing. One stator phase current of the induction machine is sampled every 15 minutes. In each 15-minute period the A/D converter records data for 10 seconds with a 6 kHz sampling frequency. The raw experimental data at two different load conditions, i.e., 50% and 33%, in [12] are used to validate the proposed fault detection algorithm. The results are plotted in Figs. 6 and 7 and compared with those obtained from the Wiener filter-based noise cancellation algorithm published in [12]. In both cases, the bearings are broken at the final stage of the experiments.

In the pretreatment of the wavelet filter, the 4th and 6th order wavelet functions Coiflet2 and Coiflet3 are selected for the cases of 50% and 33% load conditions, respectively. The Coiflet2 has the support length of 11. The Coiflet3 has the support length of 17. The parameter  $h$  in Step 3 of the proposed wavelet filter is 5000 for both cases, which is approximately 10% of the length of one batch of 25-minute current samples.

Both Figs. 6 and 7 demonstrate that the fault index, i.e., the normalized energy of the fault signal in the stator current generated by the wavelet filter, increases when the bearing condition degrades. In the 50% load case of Fig. 6(a), the result of the proposed method demonstrates that the generalized roughness of the bearing was built up during the 30th to the 50th hours of the experiment. From the 60th hour to the end of the experiment the vibrations of the fault index are kept at a high level due to the developed bearing fault. The pattern of the fault index obtained from the proposed method agrees with that obtained from the noise cancellation method in [12], which is shown in Fig. 6(b). There are two notches in the result of Fig. 6(a), which is caused by the sudden change of the total energy of the stator current. In the 33% case of Fig. 7(a), the bearing fault was built up between the 10th and 35th hour. From the 40th hour to the end of the experiment the wavelet-filtered fault index becomes unstable, because of the degraded physical condition of the bearing.

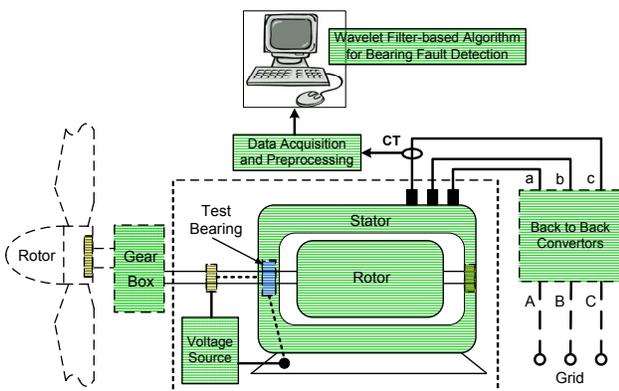


Fig. 5. Configuration of the experiment setup for incipient bearing fault detection.

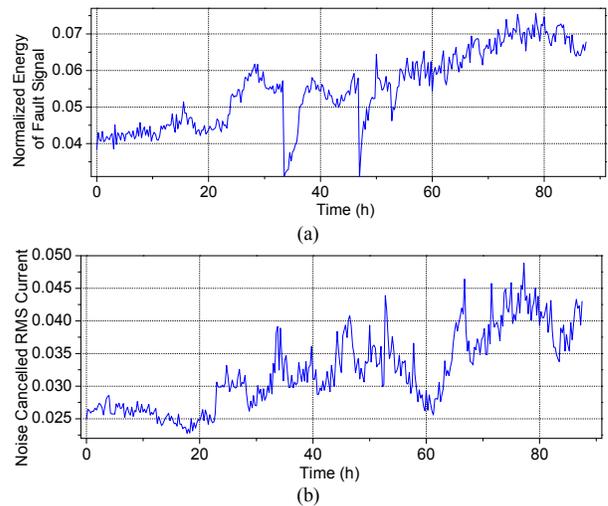


Fig. 6. At 50% load condition: (a) the result of the proposed wavelet filter-based method; (b) the result of the noise cancellation method in [12].

The wavelet filtered result gives the similar pattern to the noise-cancelled result [12] in Fig. 7(b), without the peak at the beginning of the experiment. The correlations between the proposed wavelet filter-based results and the noise-cancelled results of the 50% load case and the 33% load case are 0.69 and 0.39, respectively.

#### IV. CONCLUSIONS

A novel wavelet filter-based algorithm has been developed for detecting bearing generalized roughness faults for electric machines using stator current measurements. The method decomposes the stator current by using the DWT. The fault-related components in the stator current are located in the low energy part of the decomposed sequence due to the subtle and broad-band features of these components. The low energy

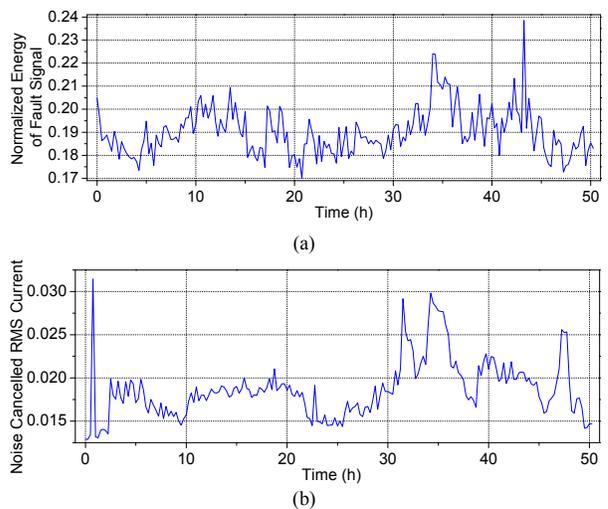


Fig. 7. At 33% load condition: (a) the result of the proposed wavelet filter-based method; (b) the result of the noise cancellation method in [12].

points of the decomposed sequence are then identified and added together as the index of the bearing faults by using the proposed wavelet filter. Experimental data have been obtained from an induction machine with developed bearing generalized roughness faults at two different load conditions. These data have been used by the proposed method for bearing fault detection. The results have been compared with those obtained from a Wiener filter-based noise cancellation algorithm presented in [12] and have shown that the proposed method is effective for incipient bearing fault detection.

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